
Groundwater Hydrology - Homework 3

Due date: Jan. 28, 2012

1. Use Theis equation (employing with the Cooper Jacob approximation) to plot the drawdown profile in a confined aquifer of transmissivity $T = 2000 \text{ m}^2/\text{day}$ and storativity $S = 10^{-3}$, where a well is pumped at the rate of $400 \text{ m}^3/\text{day}$ for 12 hours. How would the drawdown profile change if the transmissivity of the aquifer were higher? How would it change if the storativity were higher?

Solution: Theis solution is written as:

$$s(r, t) = \frac{Q}{4\pi T} W(u)$$

or using the Cooper Jacob approximation:

$$s(r, t) = \frac{Q}{4\pi T} \ln \left(2.25 \frac{Tt}{Sr^2} \right)$$

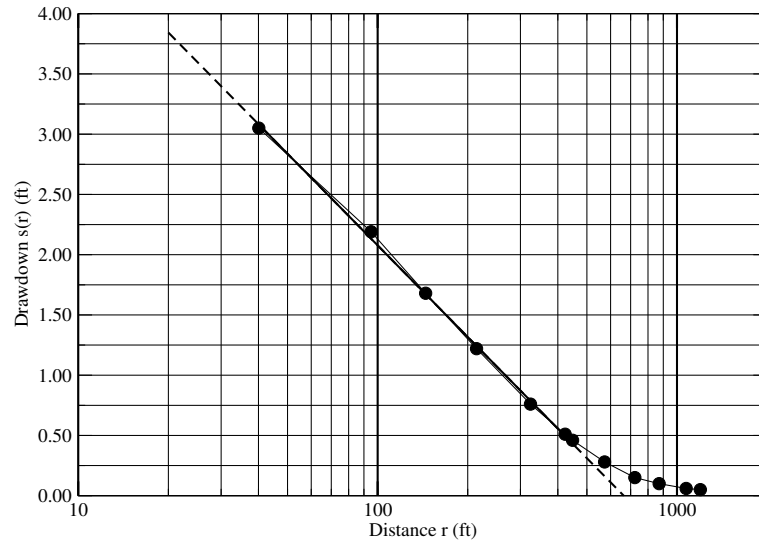
If we fix time $t = t^*$ then we can use the properties of the logarithms to find:

$$s(r, t^*) = \frac{Q}{4\pi T} \left[\ln \left(2.25 \frac{Tt}{S} \right) - 2 \ln r \right]$$

Thus, changing S or t we obtain a shift of the curve, while changing Q or T we obtain both a shift and a change in slope.

2. A pumping test is performed in a confined aquifer of thickness 100 m. The data reported in the following table and shown in the plot are collected at different distances from the pumping well after 48 hours of continuous pumping at a rate of 1500 l/min. Calculate the hydraulic conductivity K and the elastic storage coefficient S_s of the aquifer.

r (m)	s (m)
40.1	3.05
95.1	2.19
144.7	1.68
214	1.22
324	0.76
423	0.51
448	0.46
573	0.28
723	0.15
872	0.1
1073	0.06
1197	0.05



Solution The regression line has equation:

$$y = -1.1022 \ln x + 7.1538$$

Cooper Jacob tells us that:

$$s(r) = \frac{Q}{5\pi T} \ln\left(2.25 \frac{Tt}{Sr^2}\right)$$

or

$$s(r, t^*) = \frac{Q}{4\pi T} \left[\ln\left(2.25 \frac{Tt^*}{S}\right) - 2 \ln r \right]$$

Using only the first 6 points, the regression line has equation:

$$y = -1.0975 \ln(x) + 7.1311$$

thus, with $Q = 1500 \text{ l/min} = 1.5 \text{ m}^3/\text{min} = 0.025 \text{ m}^3/\text{s}$, we can calculate:

$$T = 2 \frac{Q}{4\pi 1.0975} = 3.626 \times 10^{-3} \text{ m}^2/\text{s}$$

and thus

$$K = 3.626 \times 10^{-3} / 100 = 3.626 \times 10^{-5} \text{ m/s}$$

From the line we can calculate:

$$y = 0 : \ln(x) = 7.1311 / 1.0975$$

from which $r_0 = 663.5 \text{ m}$, which is the distance at which $s = 0$, i.e.:

$$2.25 \frac{Tt}{Sr^2} = 1$$

from which:

$$S = 2.25 \frac{Tt}{r^2} = 3.202 \times 10^{-3}$$

that is $S_s = 3.202 \times 10^{-5}$ 1/m.

3. A pumping test is performed in a confined aquifer of thickness 150 m with a constant flowrate of 5000 m³/day. The data reported in the following table were collected at different times from a piezometer located 50 m from the pumping well. Using Theis equation, calculate the hydraulic conductivity K and the elastic storage coefficient S_s of the aquifer.

Note: It is impossible to fit the entire data set on the Theis type curve, so that only the first few data can be fitted correctly, as shown in Ex 4

t (days)	s (m)
1.30E-04	0
3.47E-04	0
6.94E-04	0.006
1.39E-03	0.043
3.47E-03	0.168
6.94E-03	0.302
1.39E-02	0.445
3.47E-02	0.594
6.94E-02	0.634
1.39E-01	0.637
3.47E-01	0.637
6.94E-01	0.64
2.30E+00	0.643

Solution The table from the TheisType.xls spreadsheet is:

Time (day)	Time (s)	Drawdown (m)	Time/r ²
0.00013	11.232	0	0.0044928
0.000347	29.9808	0	0.01199232
0.000694	59.9616	0.0018288	0.02398464
0.00139	120.096	0.0131064	0.0480384
0.00347	299.808	0.0512064	0.1199232
0.00694	599.616	0.0920496	0.2398464
0.0139	1200.96	0.135636	0.480384
0.0347	2998.08	0.1810512	1.199232
0.0694	5996.16	0.1932432	2.398464
0.139	12009.6	0.1941576	4.80384
0.347	29980.8	0.1941576	11.99232
0.694	59961.6	0.195072	23.98464
2.3	198720	0.1959864	79.488

$$Q \text{ (m}^3/\text{s)} = 0.05787037$$

$$R \text{ (m)} = 50$$

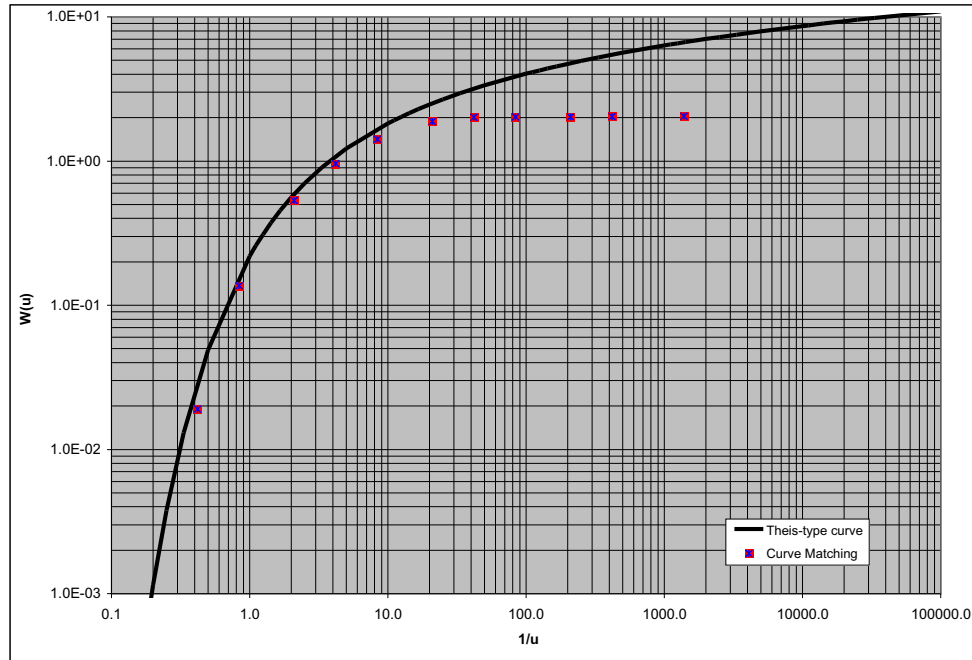
$$T \text{ (m}^2/\text{s)} = 0.048$$

$$S \text{ (/)} = 0.011$$

$$PI = 3.141592654$$

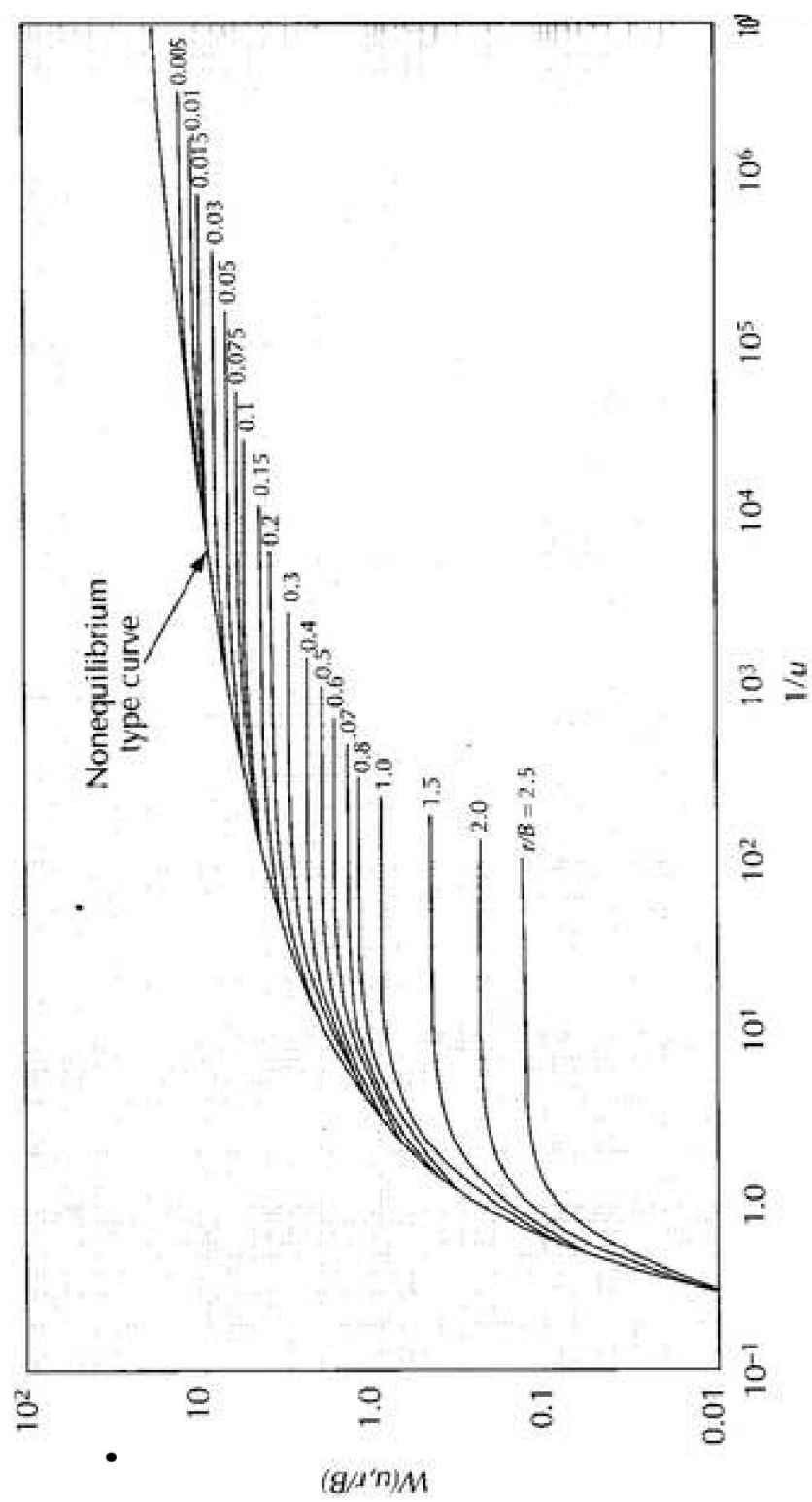
The figure is the one reported on ex. 4.

4. The following figure shows the data reported in Ex. 3 superimposed on Theis well function $W(u)$.



Explain what could cause the deviation from the Theis behavior and what is the approach that needs to be used to evaluate the characteristic constants of this phenomenon. Evaluate (approximately) these constants using the type curves reported in the figure in the next page.

Solution From the figure we can see that there is a divergence from Theis curve behavior after the 4th data point. This is due to a leaky aquifer. The horizontal behavior of the data reaches a value $W(u) = 2$. Thus we can go to the plot of Hantush type curves and find that $r/B \approx 0.55$. From this we find $B = 0.55r = 27.5$ m.



▲ FIGURE 5.11
Type curves of leaky artesian aquifer in which no water is released from storage in the confining layer. Source: W. C. Walton, Illinois State Water Survey Bulletin 49, 1962.