Groundwater Hydrology - Homework 1

Due date: Nov. 7, 2012

1. The following questions are related to particle density (defined as $\rho_s = m_s/V_s$ where m_s is the mass of grains and V_s the volume occupied by the grains) and dry bulk density ($\rho_d = m_s/V$ where V is total or bulk volume).

   (a) Derive a formula for determining dry bulk density if you are given porosity and particle density.

   (b) Which density ρ_d or ρ_s is greater in magnitude? Prove your answer.

   (c) What is a typical value for particle density?

   (d) Given a porosity of 0.28 and a dry bulk density of 1.90 g/cm^3, what is the particle density?

   (e) Now consider the wet bulk density, ρ_b, which includes the mass of water, or $\rho_b = (m_s + m_w)/V$, where m_w is the mass of fluid in the bulk volume. Given a wet bulk density $\rho_b = 2.18$ g/cm^3 and a dry bulk density, $\rho_d = 1.90$ g/cm^3, and assuming the material as completely saturated with water, what is the porosity?

   Solution:

   (a)

$$\rho_s = \frac{m_s}{V_s} \qquad \rho_d = \frac{m_s}{V}$$

$$\omega = \frac{V_v}{V} \Rightarrow \frac{\rho_d}{\rho_s} = \frac{V_s}{V} = \frac{V - V_v}{V} = 1 - \omega$$

or

$$\omega = 1 - \frac{\rho_d}{\rho_s} = \frac{\rho_s - \rho_d}{\rho_s}$$

or

$$\rho_d = \rho_s(1 - \omega)$$

   (b) from last equation, since $0 < \omega < 1$, $\rho_d < \rho_s$.

   (c) Typical value for particle density is $\rho_s = 2650$ kg/m^3=2.65 g/cm^3

   (d)

$$\rho_s = \rho_d/(1 - \omega) = 1.9/(1 - 0.28) = 2.638 \text{ g/cm}^3$$

   (e) For a completely saturated medium:

$$\rho_b = \frac{m_s}{V} + \frac{m_w}{V} = \rho_s(1 - \omega) + \rho_w\omega = \frac{\rho_d}{1 - \omega}(1 - \omega) + \rho_w\omega$$

   from which:

$$\omega = \frac{\rho_b - \rho_d}{\rho_w} = \frac{2.18 - 1.9}{1} = 0.28$$

2. The hydraulic conductivity of a coarse sand was measured in the laboratory and found to be 1.36×10^{-2} cm/s at 20 ^{o}C. What is the value of the intrinsic permeability? [the URL `http://www.thermexcel.com/english/tables/eau_atm.htm` contains a table listing all the water properties at standard (atmospheric) pressure.]

   Solution:

   From the listed web site, at 20^{o}C we have $\rho_w = 998.29$ kg/m^3, $\mu = 0.001003$ kg/(m s) and $K = 1.36 \times 10^{-4}$ m/s, thus:

$$k = \frac{\mu K}{\rho g} = \frac{0.00103 \cdot 1.36 \times 10^{-4}}{998.29 \cdot 9.81} = 1.4032 \times 10^{-11} \text{ m}^2$$

3. The measurement of water velocity from a well in a confined aquifer yields $v = 0.5$ m/day. The aquifer is characterized by a porosity $\omega = 0.25$ while the slope of the phreatic surface (water table) in the neighborhood of the observation well is equal to $i = 0.002$. Calculate the horizontal hydraulic conductivity coefficient K.

   Solution:

$$\omega|q| = K|\nabla h|$$

$$0.25 * 0.5 = Ki \Rightarrow K = \frac{0.25 * 0.5}{0.02} = 62.5 \text{ m/day}$$

4. Two piezometers (observation wells) are located in a unconfined aquifer on a line aligned with the flow direction at a relative distance of 1200 m. The measured piezometric levels are equal to +30 m a.m.s.l. and +20 m a.m.s.l. The horizontal impermeable bottom of the aquifer is located at an elevation of -10 m a.m.s.l.

   (a) Calculate Darcy velocity (specific discharge) assuming a uniform conductivity equal to $K = 18$ m/day.

   (b) Calculate Darcy velocity (specific discharge) assuming a conductivity $K_1 = 10$ m/day between $x = 0$ (where the well that measures $h = +20$ m is located) and $x = 400$ m, and $K_2 = 30$ m/day in the remaining part of the aquifer.

   Solution:

   (a)

$$|v| = K * \nabla h = 18 * 10/1200 = 0.15\text{m/day}$$

$$|q| = K * \bar{B} * \nabla h = 18 * 35 * 10/1200 = 5.25\text{m}^2\text{/day}$$

   (b)

$$K_h = \sum_i l_i / \sum_i \frac{l_i}{K_1} = (400 + 800)/(400/10 + 800/30) = 18 \text{ m/g}$$

   the results are the same as before.

5. An anisotropic two-dimensional aquifer is characterized by values of the hydraulic conductivity coefficient equal to $K_x = 36$ m/day and $K_y = 16$ m/day, where x is the principal direction of anisotropy. Given a piezometric gradient $i = 0.004$ along a direction forming an angle of $\pi/6$ with the $x+$ axis, determine Darcy's flux $\vec{q}$ and the angle between i and $\vec{q}$.

   Solution:

$$\begin{aligned}
i_x &= icos30^o = 0.004 * 0.8660254 = 0.0034641016 \\
i_y &= isin30^o = 0.004 * 0.5 = 0.002 \\
q_x &= -K_x i_x = -36 * 0.0034641016 = -0.12470766 \\
q_y &= -K_y i_y = -16 * 0.002 = -0.032 \\
\alpha &= 30^o - \text{atan}(\frac{i_y}{i_x}) = 15.608399
\end{aligned}$$

6. Consider an aquifer made up of three different formations (A, B, and C) having hydraulic conductivity and thickness: $K_A = 1.1 \times 10^{-4}$ m/s, $B_A = 45$ m; $K_B = 4.2 \times 10^{-4}$ m/s, $B_B = 35$ m; and $K_C = 9.1 \times 10^{-4}$ m/s, $B_C = 25$ m. Calculate the average horizontal and vertical hydraulic conductivities, assuming homogeneity and anisotropy in each of the formations.

   Solution:

   Use weighted arithmetic average for horizontal conductivity K_h and weighted harmonic average for vertical conductivity K_v to obtain:

$$K_h = \frac{\sum_i b_i K_i}{\sum_i b_i} = 4.04 \times 10^{-4} \text{ m/s}$$

$$K_h = \frac{\sum_i b_i}{\sum_i b_i / K_i} = 2.02 \times 10^{-4} \text{ m/s}$$

7. An unconfined aquifer encompassing an area of 4×10^4 km^2 experiences a head drop of 1.7 m after 5 years of pumping at 4.5 m^3/day. Determine the specific yield.

   Solution:

   The volume of water extracted during the 5 years is $V_w = 4.5 * 5 * 365 = 8212.5$ m^3. If the area is measured in km^2 then we have:

$$S_y = \frac{V_w}{A \cdot \Delta h} = \frac{8215.3}{4 \times 10^10 \cdot 1.7} = 1.2 \times 10^{-7}$$

   if the area is measured in m^2 then

$$S_y = \frac{V_w}{A \cdot \Delta h} = \frac{8215.3}{4 \times 10^4 \cdot 1.7} = 0.12$$

8. To prevent seawater intrusion in a confined coastal aquifer, the piezometric surface must be raised by 8 m in an area of 2 km$\times$ 20 km along the coast. Assuming for the aquifer an elastic storage $S = 5\times10^{-5}$, determine the water rate that needs to be injected in the formation through a line of wells parallel to the coast.

   Solution:

$$V = \Delta h * Area * S = 8 * 2 * 20 * 1000 * 1000 * 5 \times 10^{-5} = 16000 \text{ m}^3$$

9. A horizontal aquifer is capped by a 20 meter thick clayey aquitard that extends to the ground surface, where the water table is located. The clay is characterized by a dry bulk density of 1900 kg/m^3 and a porosity of 20%.

   (a) Calculate the total stress acting on the top of the aquifer.

   (b) Calculate the effective stress in the aquifer, assuming an average pressure head in it of 30 m.

   (c) Calculate the compaction caused by pumping, assuming the aquifer to have a thickness of $B = 9$ m and a vertical compressibility $c_b = 2.5 \times 10^{-8}$ m^2/N,

   (d) The porosity of the aquifer is 0.3 and its hydraulic conductivity is $K = 4.5 \times 10^{-5}$ m/s. Calculate the transmissivity ($= BK$) and storativity ($S = BS_s$) of the aquifer assuming a volumetric compressibility of water $\beta = 4.5 \times 10^{-10}$ m^2/N.

   Solution:

   (a) $\rho_b = \rho_d + \omega\rho_w = 1900 + 1000 * 0.2 = 2100$ kg/m^3 thus we can write:

   $$\sigma_T = \sigma_z + p_w = 2100 * 9.81 * 20 = 411.6 \text{ kPa}$$

   (b) $p_w = \rho_w g B = 1000 * 9.81 * 30 = 294$ kPa
   $\sigma_z = \sigma_T - p_w = 411.6 - 294 = 117.6$ kPa

   (c) $\Delta B = c_b B \Delta p$ assuming $\Delta h = 1$ m, we have
   $\Delta p = \Delta h \cdot \rho_w \cdot g = 1 * 1000 * 9.81 = 0.981 \times 10^4$ yielding:
   $\Delta B = 2.5 \times 10^{-8} * 0.981 \times 10^4 * 9 = 0.0207$ m

   (d) $T = BK = 9 * 4.5 \times 10^{-5} = 4.05 \times 10^{-5}$ m^2/s
   $S = B\rho_w g(c_b + \omega\beta) = 9 * 1000 * 9.81 * (2.5 \times 10^{-8} + 0.3 \cdot 4.5 \times 10^{-10}) = 2.21 \times 10^{-3}$

10. Water extraction from a confined aquifer having thickness z=20 m produces a pressure variation Δp= -6 kg/cm^2 and an 8% reduction of the initial void index e_0=0.3. Calculate

    (a) the vertical compressibility of the soil c_b;

    (b) the vertical deformation (compaction) Δz of the aquifer.

    Solution:

    We have:

    (a) On can express c_b in m^2/kg or in Pa. Using Pa, we have: 6 kg/m^2= 6$\cdot$ 98100 = 5.9$\times$10^5 Pa. Then
    $c_b = \frac{1}{1+e}\frac{\Delta e}{\Delta p} = \frac{1}{1+0.3}\frac{0.3*(1-0.08)}{5.9\times10^5} = 1.05 \times 10^{-7}$ Pa

    (b) $\Delta z = c_b \cdot \Delta p \cdot z = 1.05 \times 10^{-7} \cdot 5.9 \times 10^5 \cdot 20 = 1.24$ m

11. A confined aquifer is characterized by a porosity $\omega = 0.2$, an initial thickness $b_0 = 10$ m and is located at an average depth $z = 500$ m. The aquifer is subjected to a uniform pressure variation and can be assumed to be normally pressurized (hydrostatic conditions can be assumed for the entire vertical column). The following measures of the variation of aquifer thickness with pressure have been obtained by means of the radioactive marker technique:

b [m]	10	9.98	9.96
Δp [MPa]	0	2	4

Knowing that water density varies according to the following law:

$$\rho_w = \rho_0 e^{\beta(p-p_0)} \qquad \beta \approx 5 \times 10^{-10} \text{ Pa}^{-1}$$

calculate the elastic storage coefficient S_s of the aquifer.

Solution:
Note that the data points are disposed along a straight line. Then c_b is the slope of this line, i.e.:
$c_b = 1.0 \times 10^{-9}$ 1/Pa
$S_s = \rho_w g(c_b + \omega\beta) = 1000 \cdot 9.81(10^{-9} + 0.2 \cdot 4.5 \times 10^{-10}) = 1.07 \times 10^{-5}$ 1/m