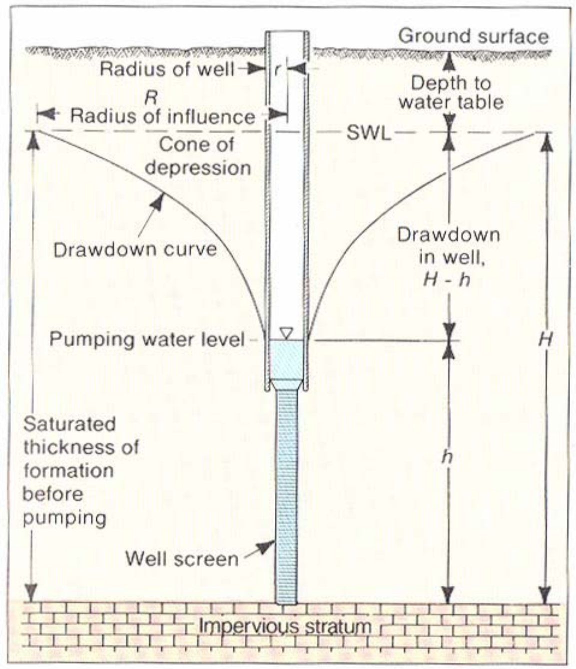
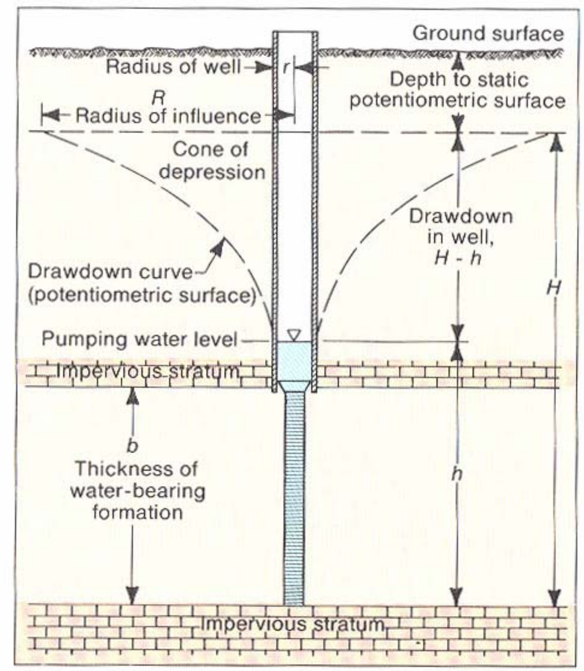
Well Hydraulics

The time required to reach steady state depends on S(torativity) T(ransmissivity) BC(boundary conditions) and Q(pumping rate).

- **cone of depression**
- **static water level (SWL)**
- **drawdown**
- **residual drawdown**
- **radius of influence**
- **storage coefficient *S***
- **transmissivity *T***



Well in an unconfined aquifer showing the meaning of the various terms used in the equilibrium equation.



Well in a confined aquifer showing the meaning of various terms used in the equilibrium equation.

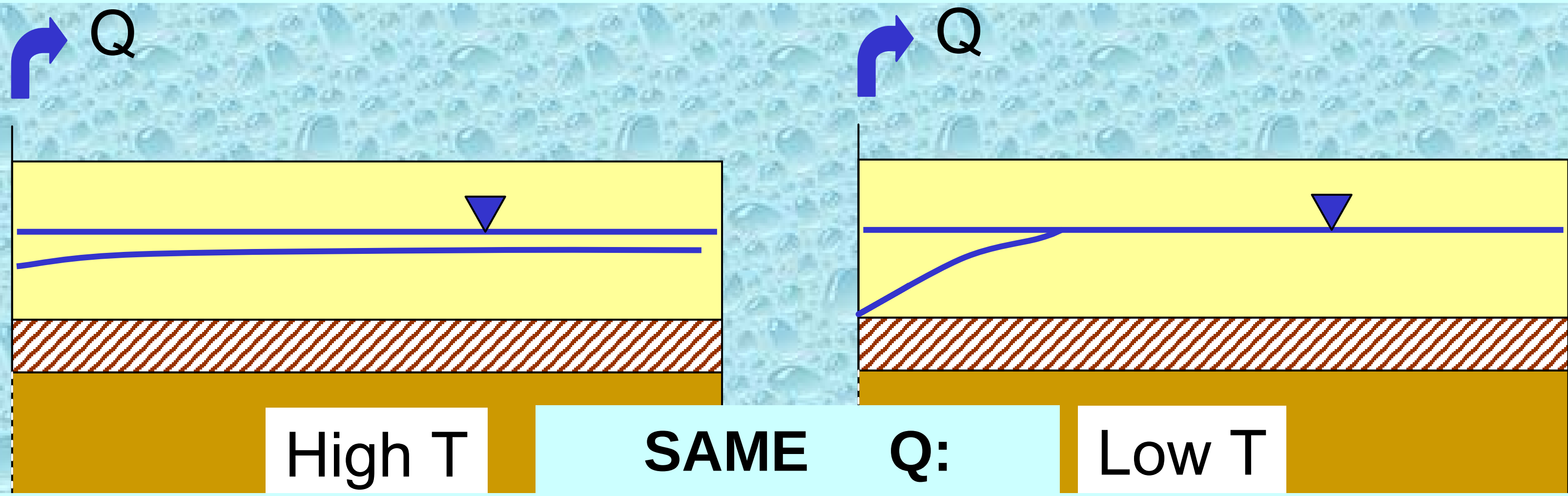
Simplest Situation

Steady State, Confined, inflow balances outflow

Only relationships between drawdown and distance need be considered

Only Transmissivity matters and can be determined ... Storage properties are irrelevant

The shape of the drawdown cone is controlled by pumping rate Q and Transmissivity,
lower T requires a higher gradient for the same Q



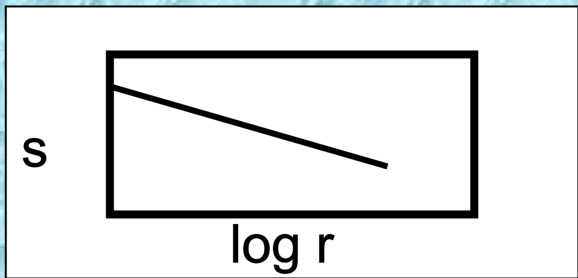
Darcy's Law Q=KiA is satisfied on every cylinder around the well, the gradient decreases linearly with distance from the well as the area increases linearly

STEADY STATE, CONFINED

Assuming
- aquifer is homogeneous, isotropic, areally infinite
- pumping well fully penetrates and receives water from the entire thickness of the aquifer
- Transmissivity is constant in space and time
- pumping has continued at a constant rate long enough for steady state to prevail
- Darcy's law is valid

Plot s vs log r from a number of wells as follows

s vs. log r - is a straight line, if assumptions are met, drawdown decreases logarithmically with distance form the well because gradient decreases linearly with increasing area ($2\pi rh$)



$$Q = \frac{2\pi T(h_2 - h_1)}{\ln(r_2/r_1)}$$

Theim Eqtn

and rearranging to get T from field data:

$$T = \frac{Q}{2\pi(h_2 - h_1)} \ln\left(\frac{r_2}{r_1}\right)$$

T = transmissivity [L^2/T]
Q = discharge from pumped well [L^3/T]
r = radial distance from the well [L]
h = head at r [L]

In an unconfined aquifer, T is not constant

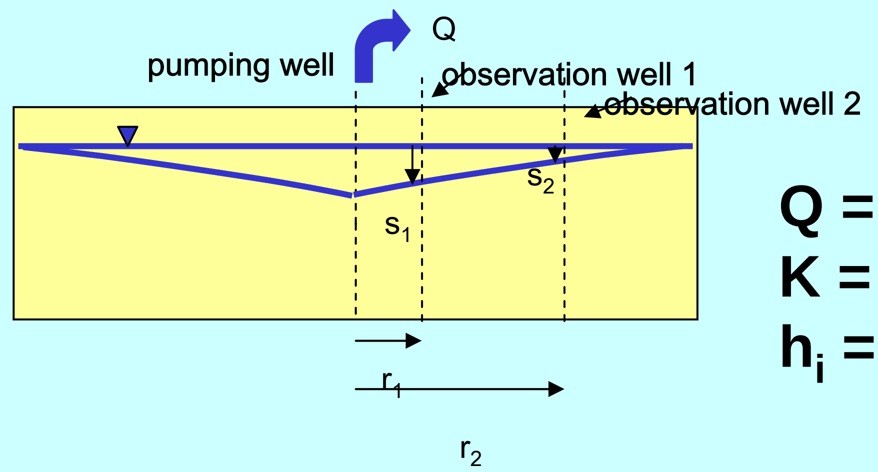
If drawdown is small relative to saturated thickness, confined equilibrium formulas can be applied with only minor errors

Otherwise call on Dupuit assumptions and use:

$$Q = \pi K \frac{(h_2^2 - h_1^2)}{\ln(r_2/r_1)}$$

or, to determine K from field measurements of head:

$$K = \frac{Q \ln\left(\frac{r_2}{r_1}\right)}{\pi (h_2^2 - h_1^2)}$$



Q = pumping rate [L^3/T]

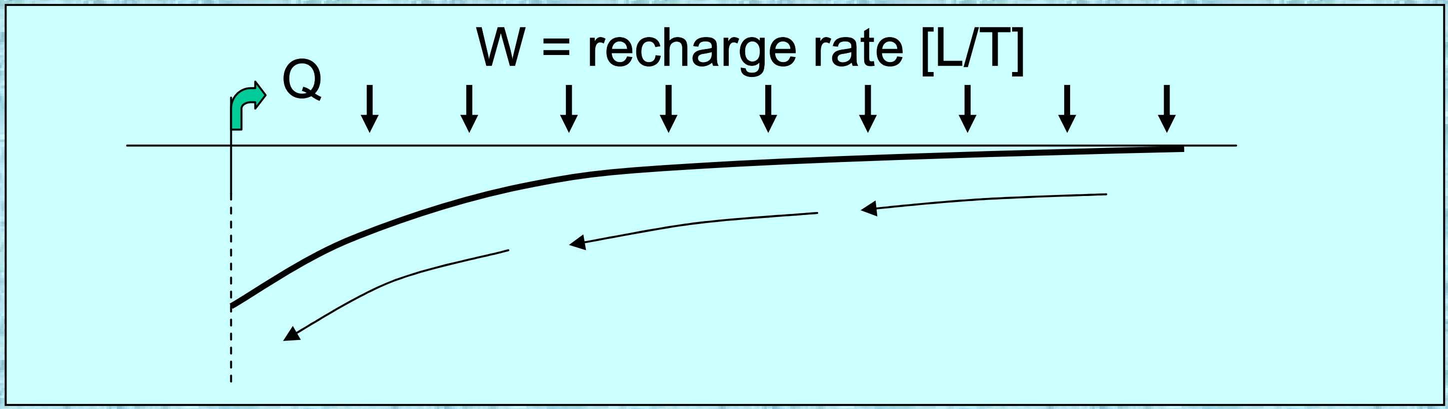
K = permeability [L/T]

h_i = head @ a distance r_i from well [L]

using the aquifer base as datum

The aquifer base must be the datum because the head not only represents the gradient but also reflects the aquifer thickness, hence the flow area.

More likely, vertical leakage will satisfy Q with w = recharge rate, then:



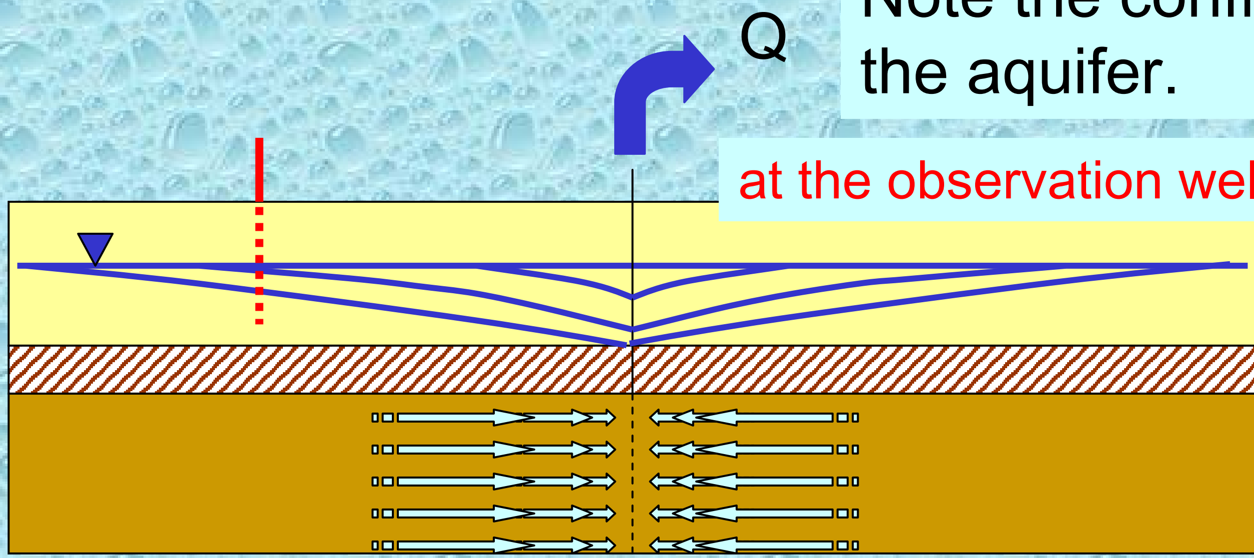
We can include recharge in the expression:

$$h_2^2 - h_1^2 = \frac{w}{2K}(r_1^2 - r_2^2) + \frac{Q}{\pi K}\ln\left(\frac{r_2}{r_1}\right)$$

and solve for K:

$$K = \frac{\frac{w}{2}(r_1^2 - r_2^2) + \frac{Q}{\pi}\ln\left(\frac{r_2}{r_1}\right)}{(h_2^2 - h_1^2)}$$

Note the confined character of the aquifer.



at the observation wells …..

C The red well will reflect the water level in the upper aquifer and we will not know about any of the blue water levels shown in the diagram.

A The aquifer is confined because water levels in the bore that penetrates the aquifer are above the top of the aquifer.

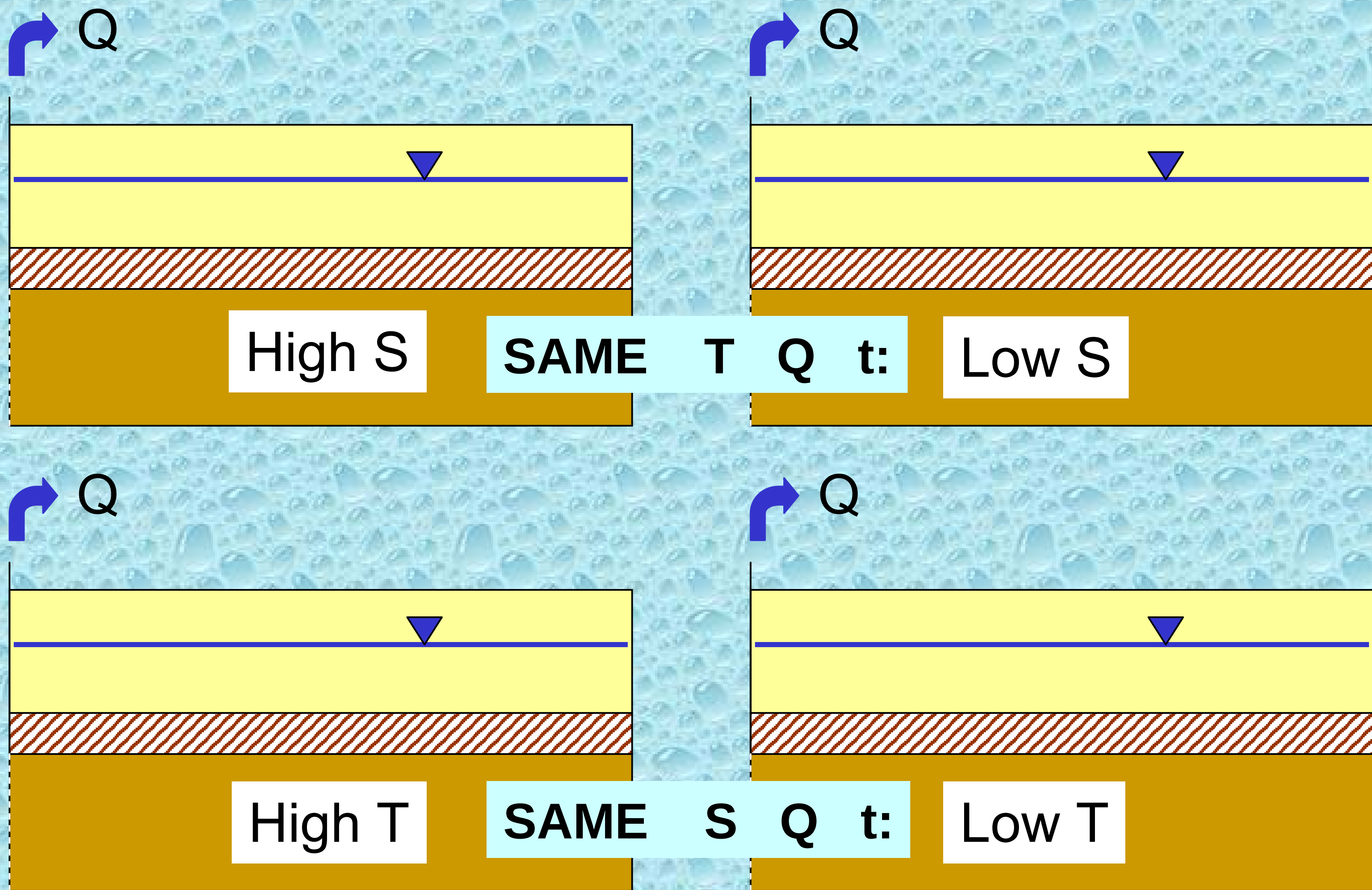
The Qualitative Viewpoint:

Infinite Aquifer, Initially hydrostatic

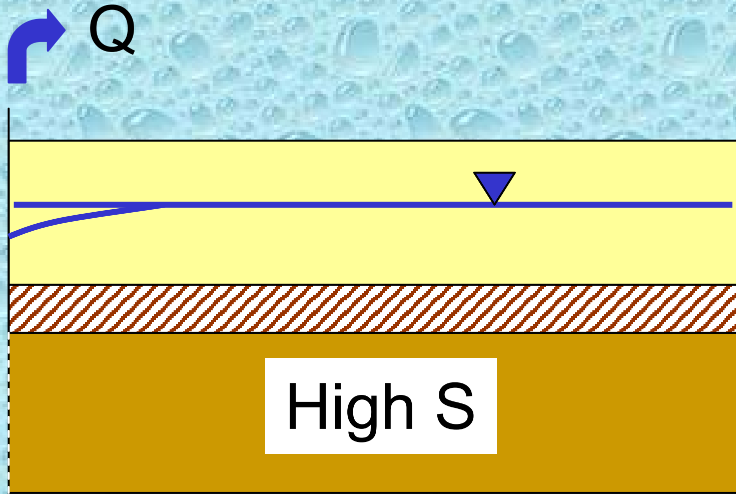
Water flows "more easily" in high T material vs low T, Thus for the same Q
steeper gradients occur in low T material

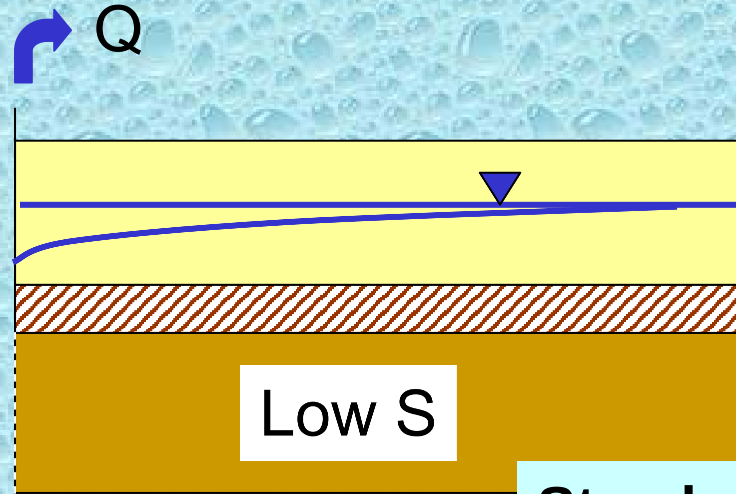
Initially water is removed from storage near the well bore
If S is high: we get more water
for the same drop in head
over the same area
compared with low S

Sketch the relative drawdown cones for the cases below



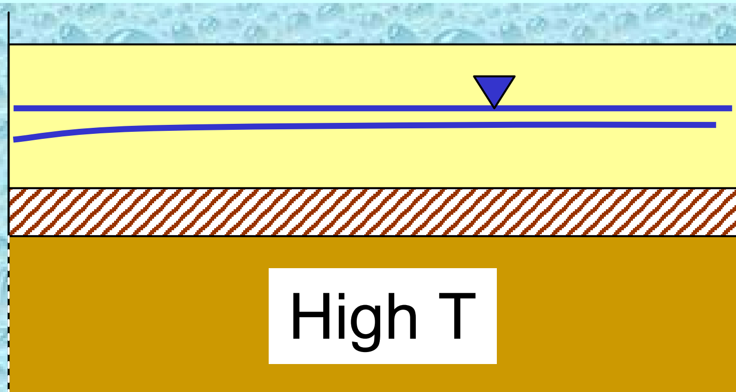
SAME T Q t :

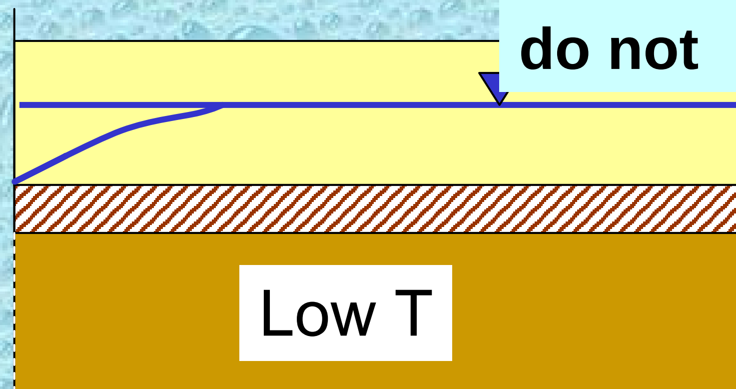




same T yields same gradient at well bore
different S yields different volume drawdown cone
smaller S yields proportionally larger cone

Steady state:
T differences persist
S differences do not





SAME S Q t :

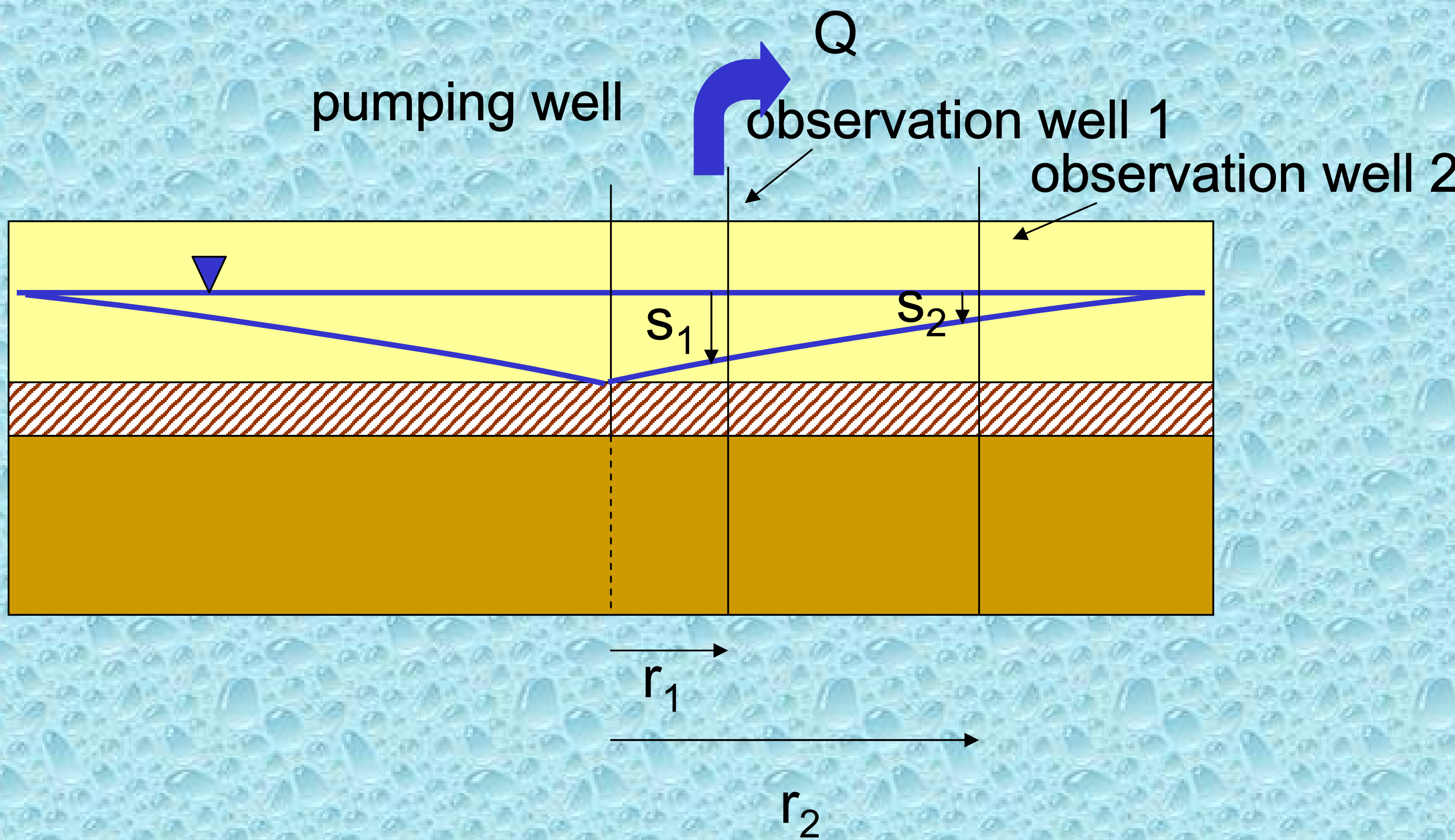
USING TRANSIENT RESPONSE OF SYSTEMS TO PUMPING

Benefits/Disadvantages

- can estimate storativity values
- when estimating aquifer properties, we get results at early time
- easier to detect extraneous effects
- analysis is more complex

Assumptions

1. homogeneous, isotropic, infinite areal extent
2. pumping well fully penetrates and receives water from the entire thickness of the aquifer
3. Transmissivity is constant in space and time
4. well has infinitesimal diameter
5. water removed from storage is discharged instantaneously with decline in head



Q
pumping well
observation well 1
observation well 2
s1
s2
r1
r2

Theis Equation

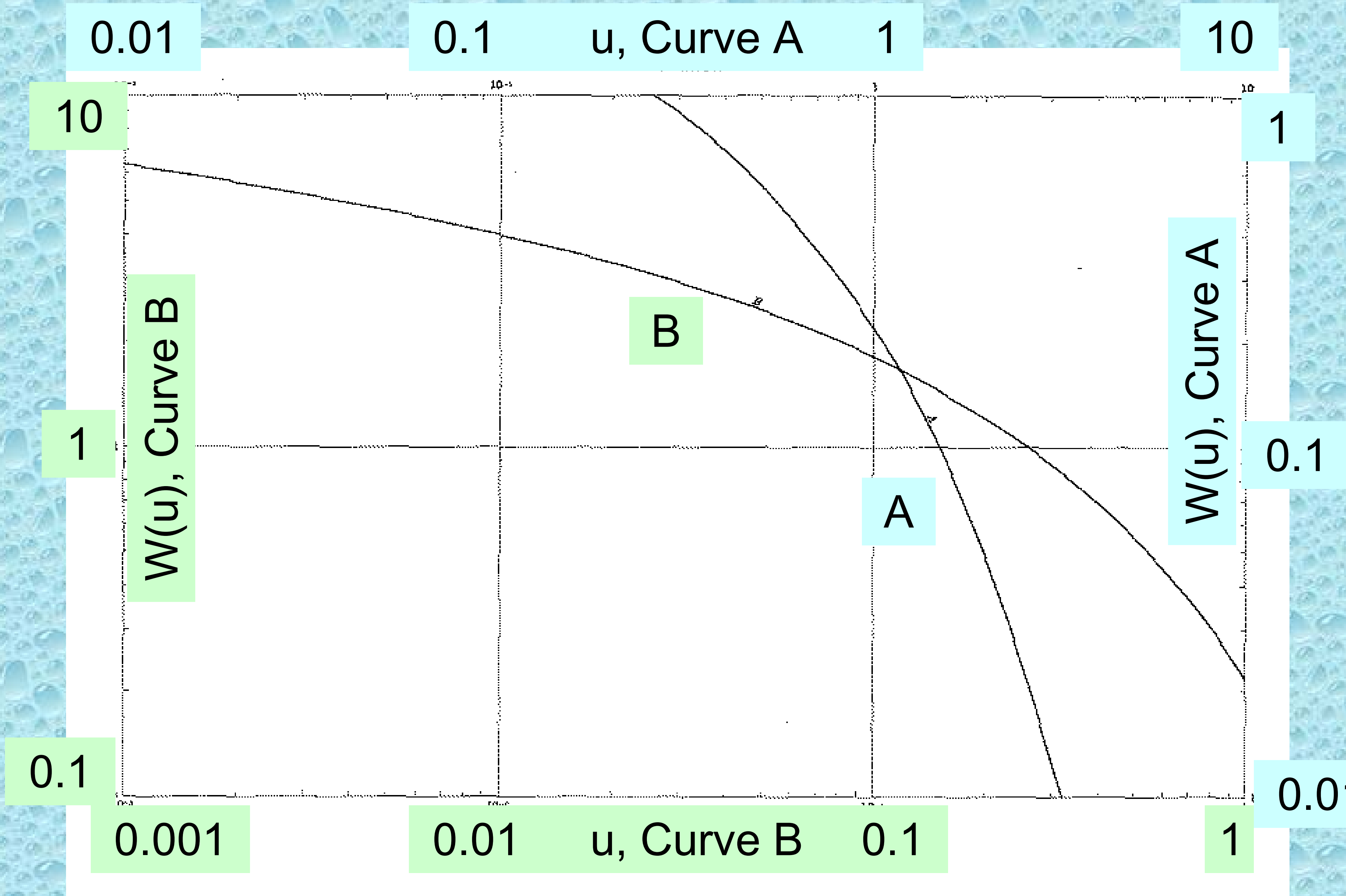
$$s = h_o - h = \frac{Q}{4\pi T} W(u)$$

$$u = \frac{r^2 S}{4Tt} \quad \text{or} \quad \frac{r^2}{t} = \frac{4T}{S} u$$

s = drawdown [L]
h_o = initial head @ r [L]
h = head at r at time t [L]
t = time since pumping began [T]
r = distance from pumping well [L]
Q = discharge rate [L^3/T]
T = transmissivity [L^2/T]
S = Storativity [ ]

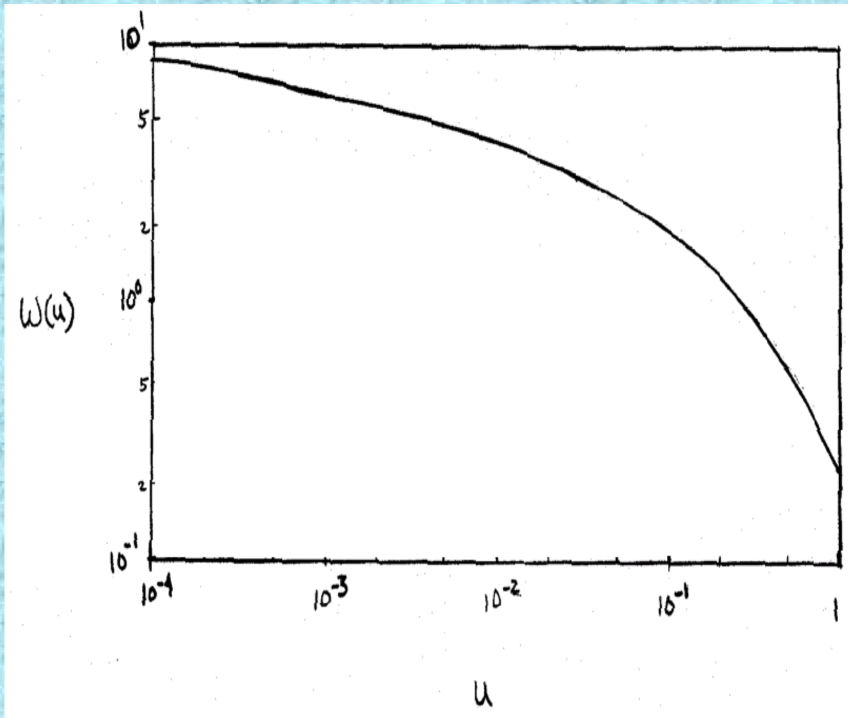
$$W(u) = \int_u^\infty \frac{e^{-u}}{u} du = [-0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \ldots\ldots]$$

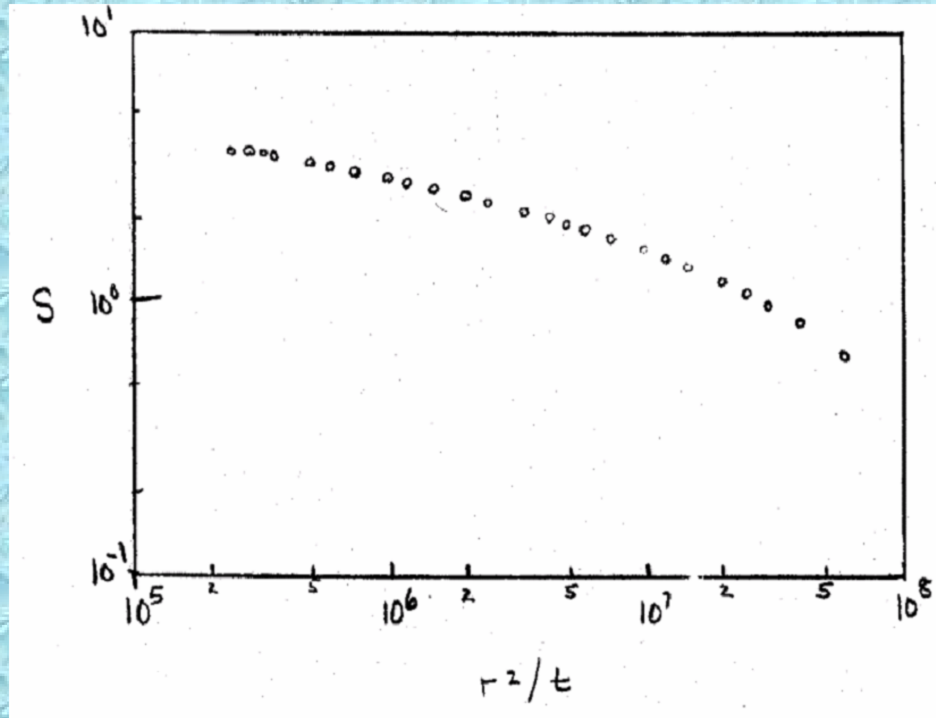
u	$W(u)$	u	$W(u)$	u	$W(u)$	u	$W(u)$
1×10^{-10}	22.45	7×10^{-8}	15.90	4×10^{-5}	9.55	1×10^{-2}	4.04
2	21.76	8	15.76	5	9.33	2	3.35
3	21.35	9	15.65	6	9.14	3	2.96
4	21.06	1×10^{-7}	15.54	7	8.99	4	2.68
5	20.84	2	14.85	8	8.86	5	2.47
6	20.66	3	14.44	9	8.74	6	2.30
7	20.50	4	14.15	1×10^{-4}	8.63	7	2.15
8	20.37	5	13.93	2	7.94	8	2.03
9	20.25	6	13.75	3	7.53	9	1.92
1×10^{-9}	20.15	7	13.60	4	7.25	1×10^{-1}	1.823
2	19.45	8	13.46	5	7.02	2	1.223
3	19.05	9	13.34	6	6.84	3	0.906
4	18.76	1×10^{-6}	13.24	7	6.69	4	0.702
5	18.54	2	12.55	8	6.55	5	0.560
6	18.35	3	12.14	9	6.44	6	0.454
7	18.20	4	11.85	1×10^{-3}	6.33	7	0.374
8	18.07	5	11.63	2	5.64	8	0.311
9	17.95	6	11.45	3	5.23	9	0.260
1×10^{-8}	17.84	7	11.29	4	4.95	1×10^{0}	0.219
2	17.15	8	11.16	5	4.73	2	0.049
3	16.74	9	11.04	6	4.54	3	0.013
4	16.46	1×10^{-5}	10.94	7	4.39	4	0.004
5	16.23	2	10.24	8	4.26	5	0.001
6	16.05	3	9.84	9	4.14		

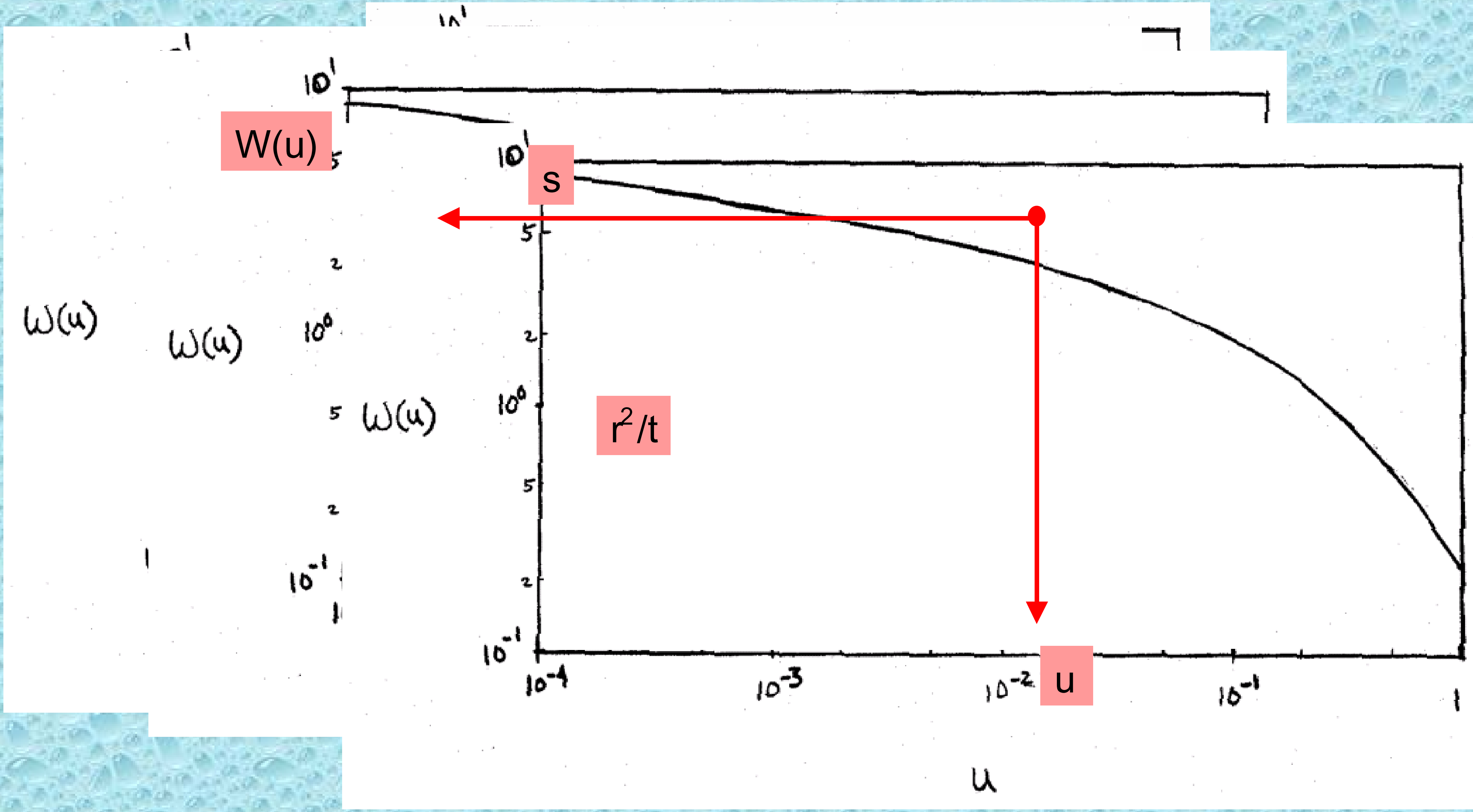

0.01
0.1
u, Curve A
1
10
10
1
W(u), Curve B
B
1
A
W(u), Curve A
0.1
0.1
0.01
0.001
0.01
u, Curve B
0.1
1

Use of the Theis equation to determine T and S from a pumping test

matching the W(u) curve on a log-log plot s vs r^2/t







$$T = \frac{1}{4\pi}\frac{Q}{s}W(u)$$

$$S = \frac{4uTt}{r^2}$$

Cooper-Jacob simplified the Theis equation for large t and small r using only the first 2 terms of the W(u) function

$$s = h_o - h = \frac{Q}{4\pi T} W(u)$$

$$u = \frac{r^2 S}{4Tt}$$

$$W(u) = \int_u^\infty \frac{e^{-u}}{u} du = [-0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \ldots\ldots]$$

s = drawdown [L]

h_o = initial head @ r [L]

h = head at r at time t [L]

t = time since pumping began [T]

r = distance from pumping well [L]

Q = discharge rate [L^3/T]

T = transmissivity [L^2/T]

S = Storativity [ ]

What is small?

$u < 0.01$

$$s = \frac{2.3Q}{4\pi T} \log \frac{2.25Tt}{r^2 S}$$

Simplified expression

$$s = \frac{2.3Q}{4\pi T} \log \frac{2.25Tt}{r^2 S}$$

Since Q r T & S are constant, s vs log t should be a straight line then:

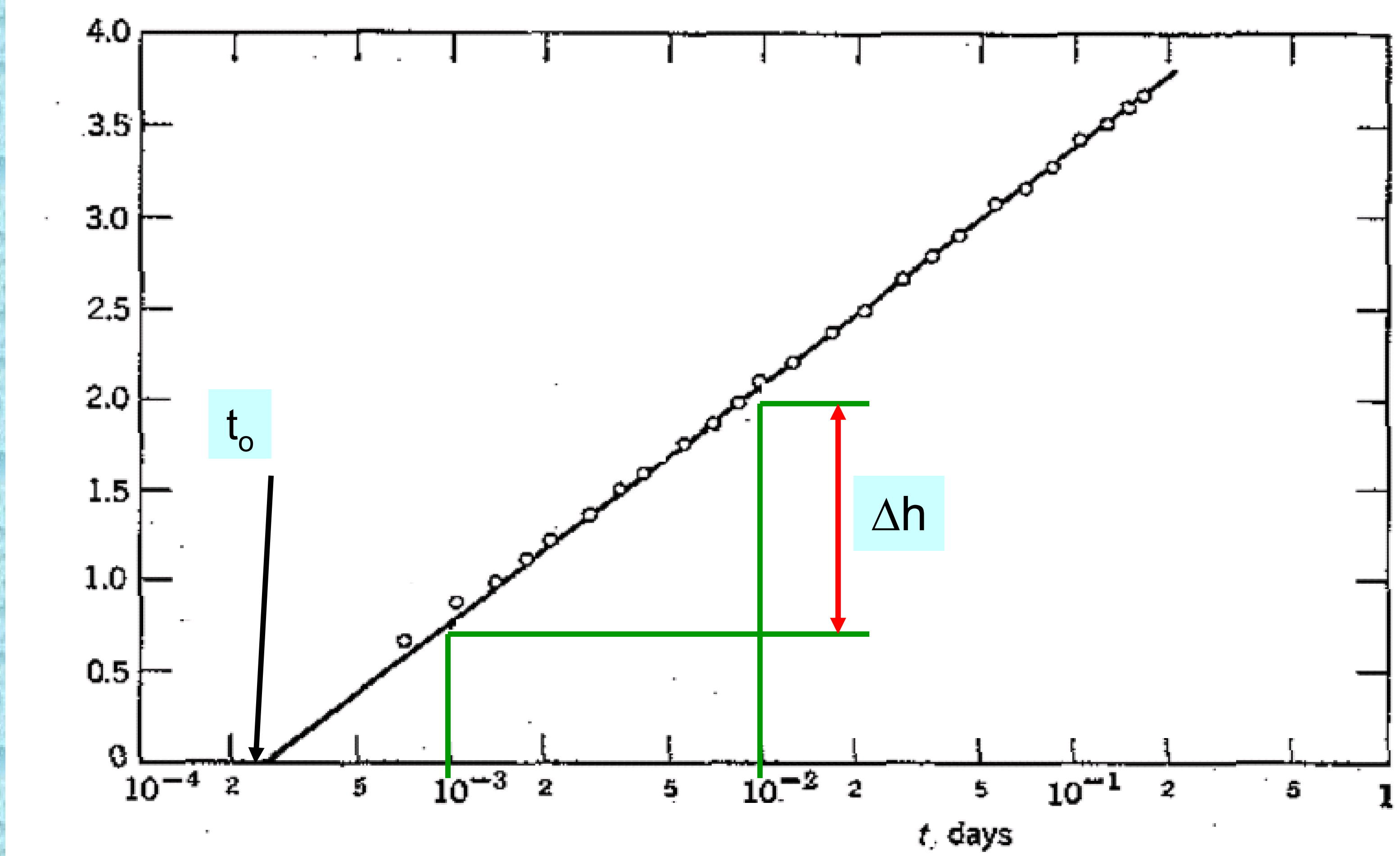
$$T = \frac{2.3Q}{4\pi \Delta h}$$

$$S = \frac{2.25Tt_o}{r^2}$$

where:

Δh = drawdown over 1 log cycle of time

t_o = time intercept for zero drawdown

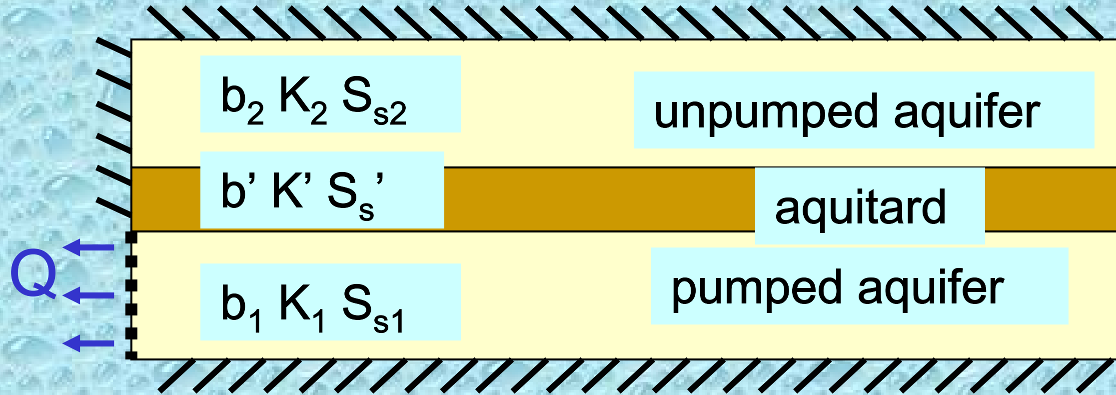


Plot <u>Δh vs log t</u> (for one point in time) and use:

Δh drawdown over 1 log cycle

t_o = time corresponding to Δh=0

LEAKY AQUIFERS

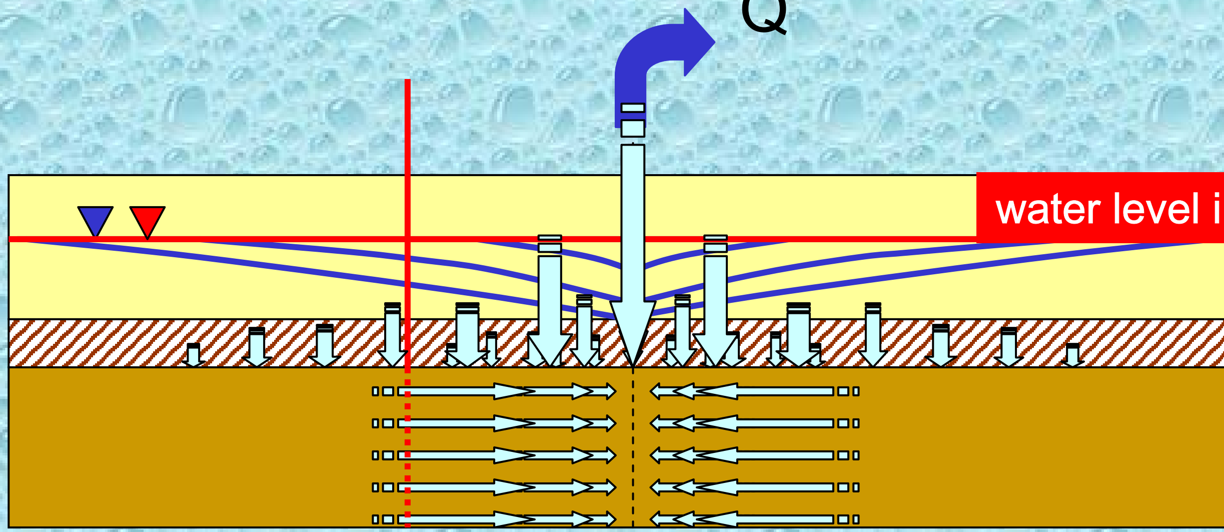


previously K' was zero (i.e. no leakage)

subscript 1 = pumped zone
subscript 2 = unpumped aquifer
prime = aquitard
Q = pumping rate
b = thickness
K = hydraulic conductivity
S_s = specific storage

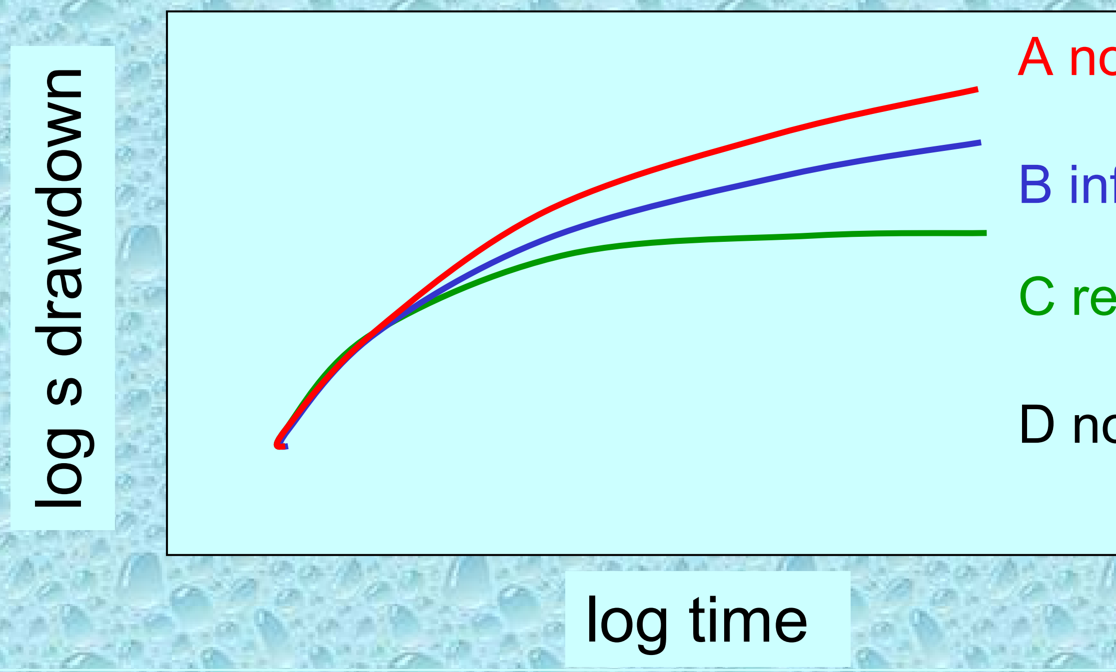
assume:
no head change in shallow aquifer
horizontal flow in aquifers
vertical flow in aquitards
aquifer extends far enough to intercept enough leakage to satisfy Q
Theis assumptions (other than infinite aquifer) apply

Solution is similar to Theis Solution but well function is more complex



Transients will occur as the head decrease moves up through the aquitard. S and Kv of the aquitard will control the rate of progress.

will the red observation well look more like? …..



A no-flow boundary

B infinite aquifer

C recharge boundary

D none of the above

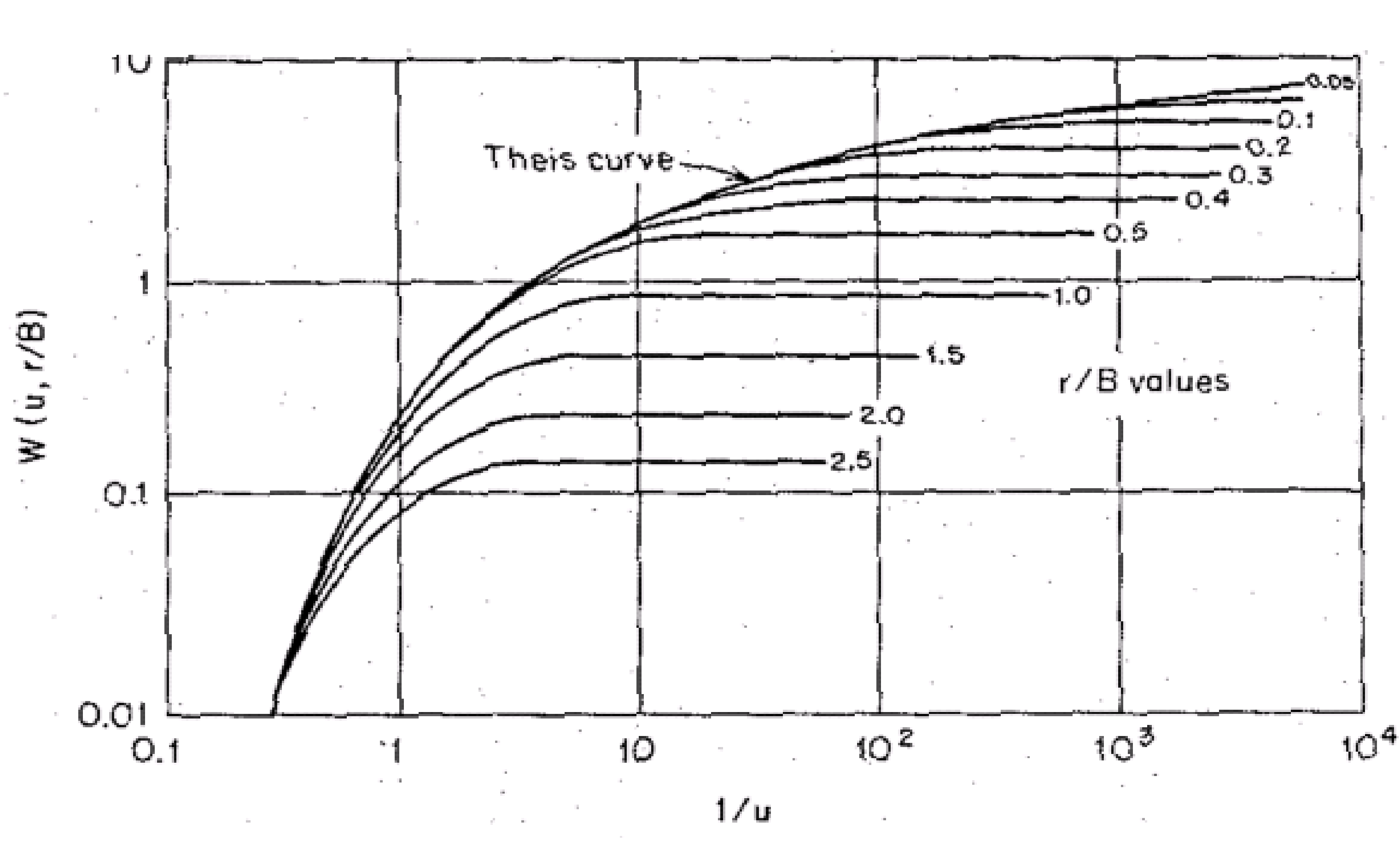
Hantush & Jacob 1955

assumed no storage in the aquitard

and expressed this solution

in terms of a dimensionless parameter

$$\frac{r}{B} = r\sqrt{\frac{K'}{K_1 b_1 b'}}$$



Curve match
$W(u, r/B)$ r/B $1/u$ matched with s t

$$s = h_o - h = \frac{Q}{4\pi T} W(u, r/B)$$

Solve for K_1 from

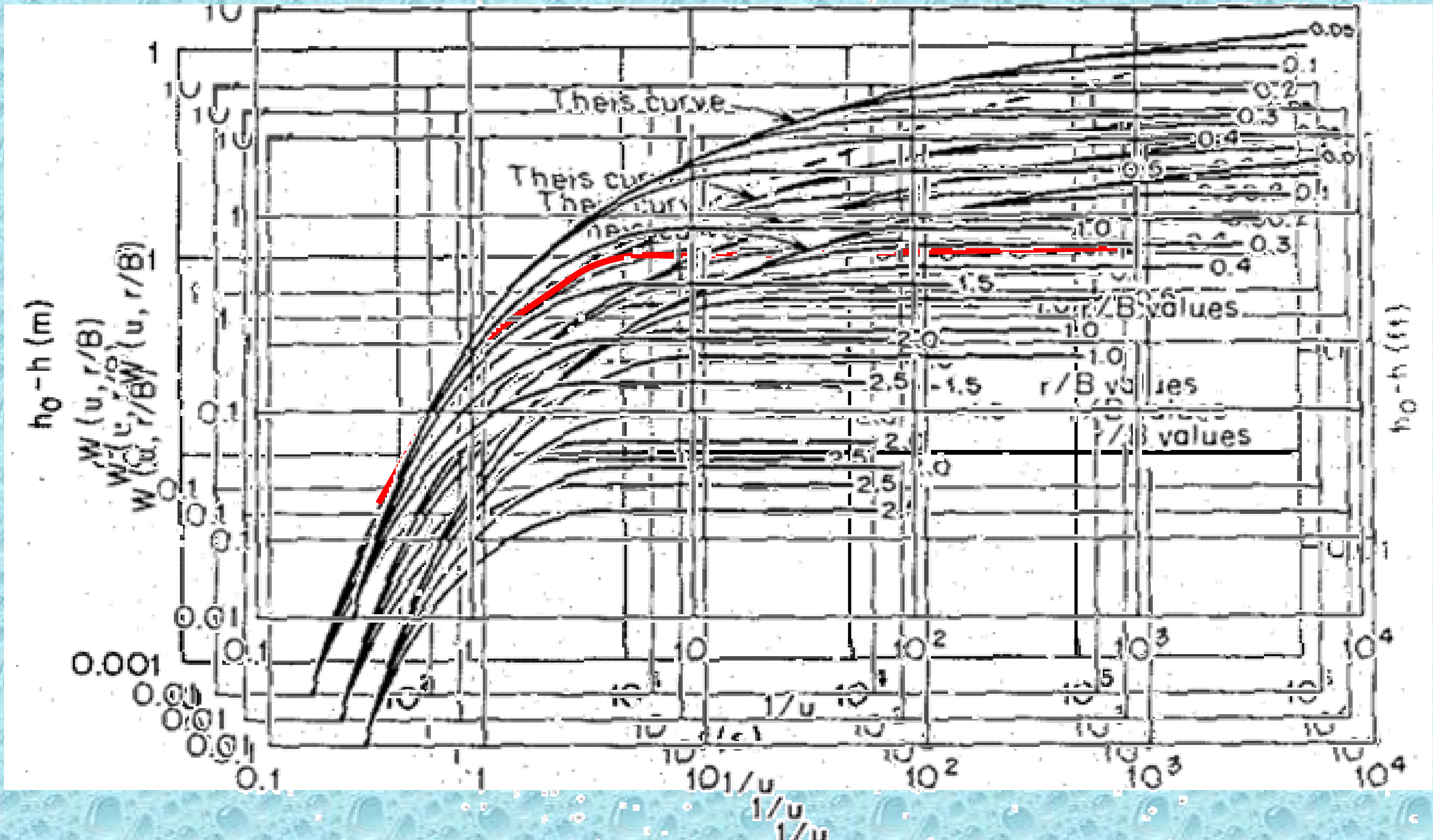
$$K_1 = T/b_1$$

Solve for K’ from

$$\frac{r}{B} = r\sqrt{\frac{K'}{K_1 b_1 b'}}$$

Solve for S_1 from

$$u = \frac{r^2 S}{4Tt}$$

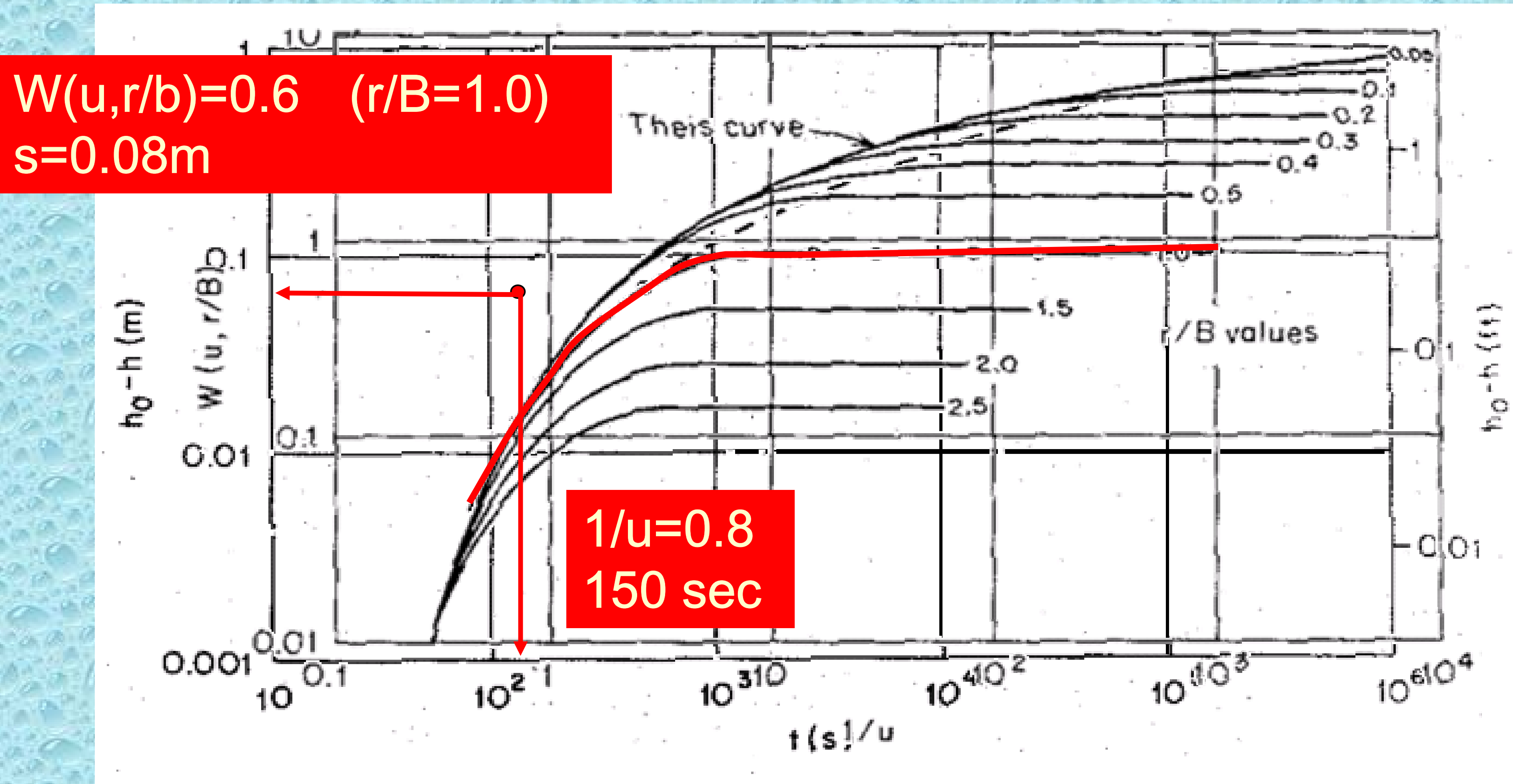


$Q=0.004\ m^3/sec$

$r=55m$

$b_1=30.5m$

$b'=3.05m$



$Q=0.004\ m^3/sec$ $b_1=30.5m$

$r=55m$ $b'=3.05m$

SUPERPOSITION APPLICATIONS

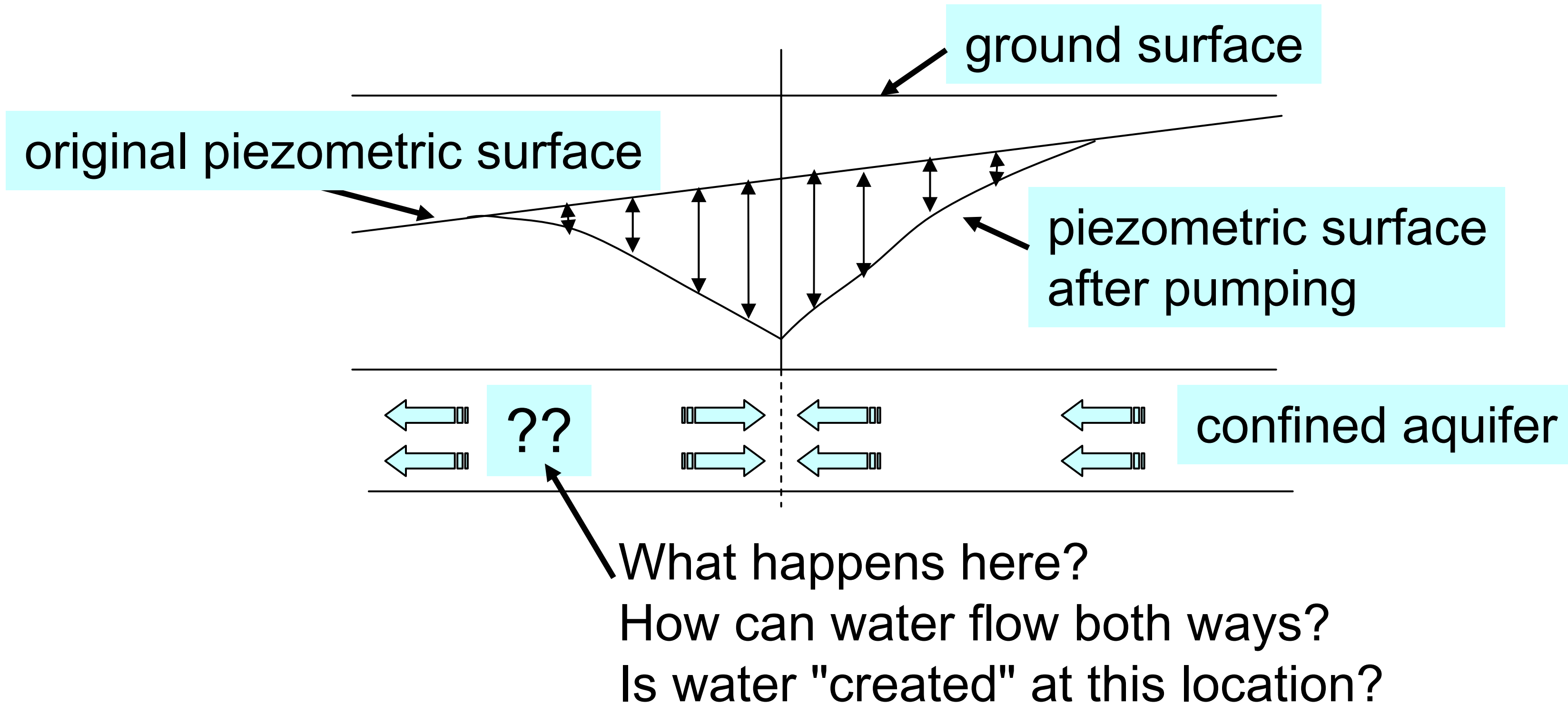
applicable to linear conditions
(i.e. confined or unconfined if drawdown << aquifer thickness s<<b)

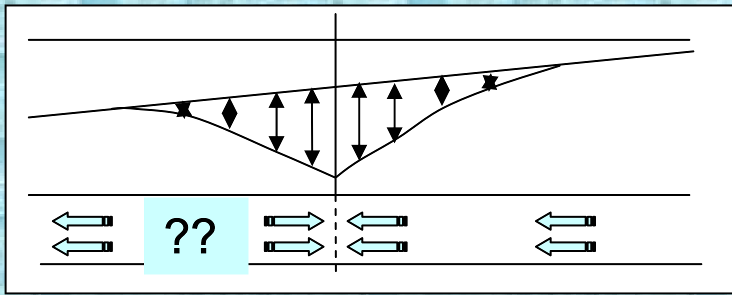
Utility of superposition

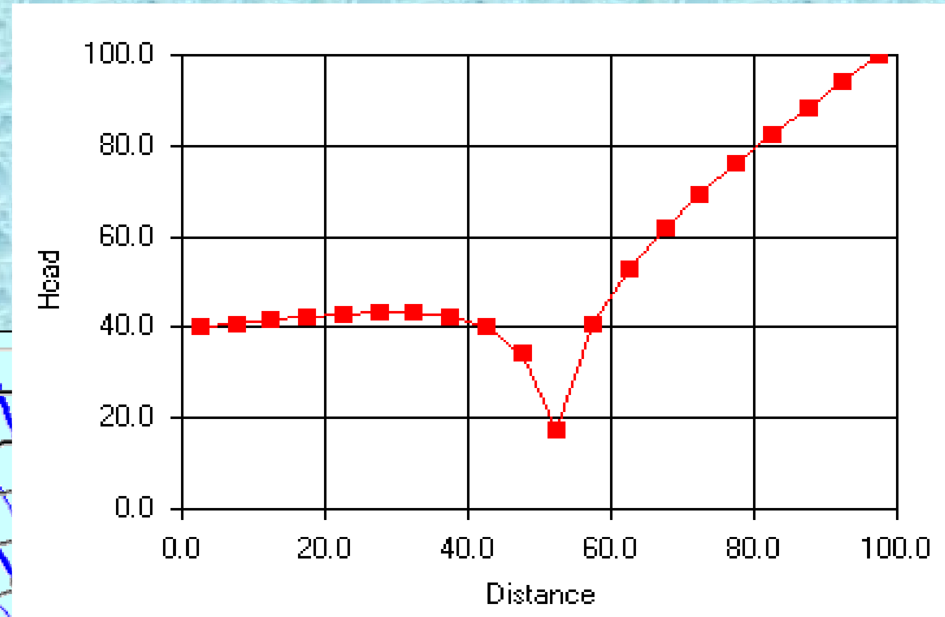
- Impact of pumping on a flow field
- Image well theory
- Pumping from a number of wells
- Incremental Pumping
- Aquifer test recovery data

IMPACT OF PUMPING ON A FLOW FIELD

Drawdown from pumping is superimposed on the initial flow field (i.e. subtract drawdown from undisturbed piezometric head)







The area is surrounded by contours 42 and 44, thus heads must ie between these values.

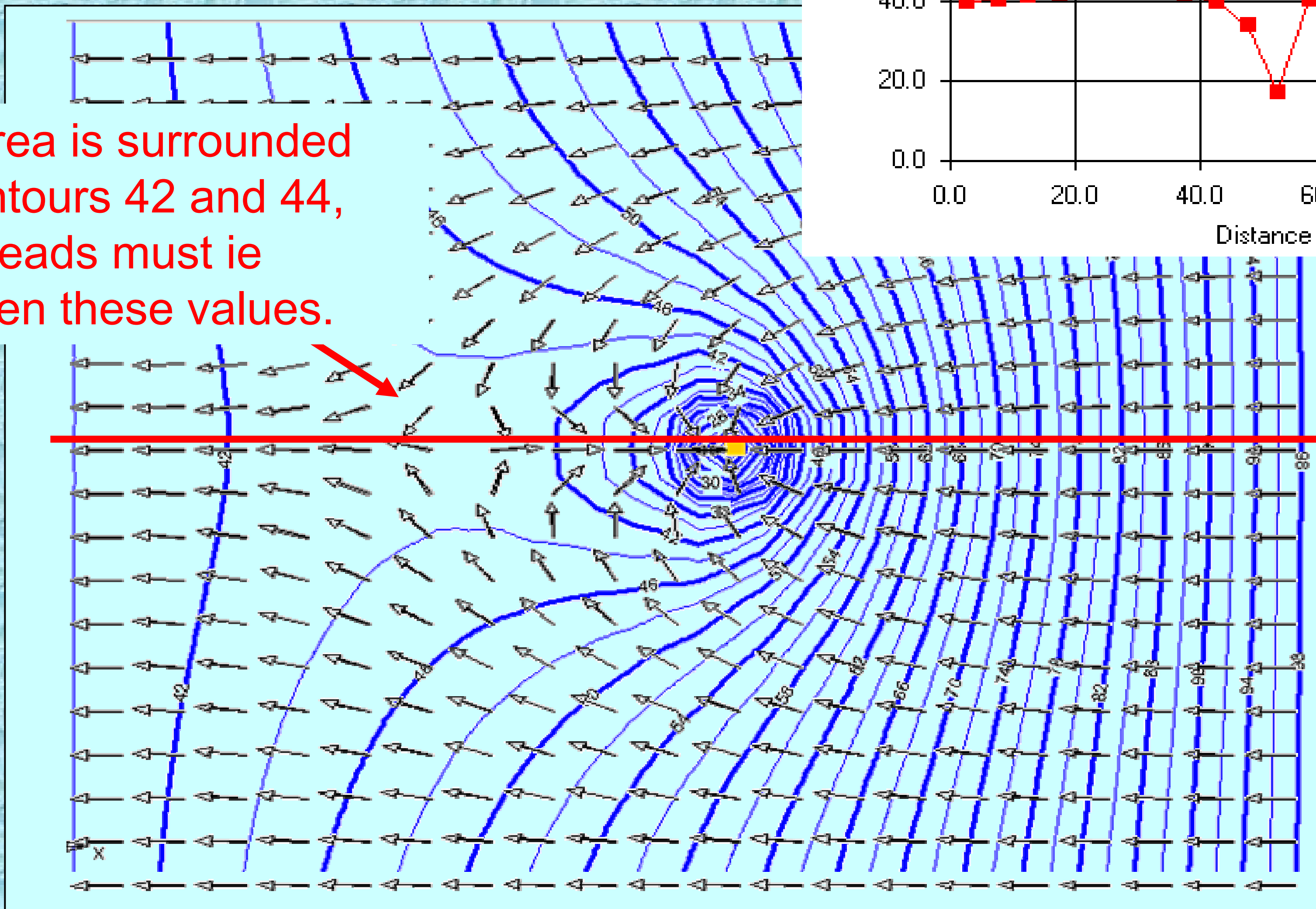
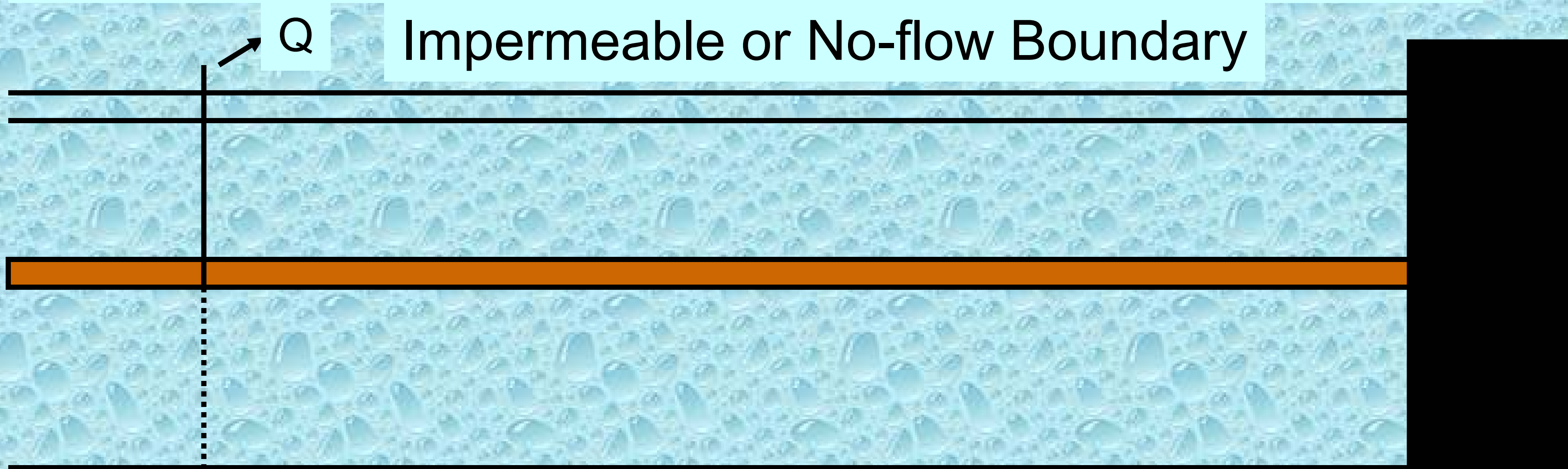


IMAGE WELL THEORY

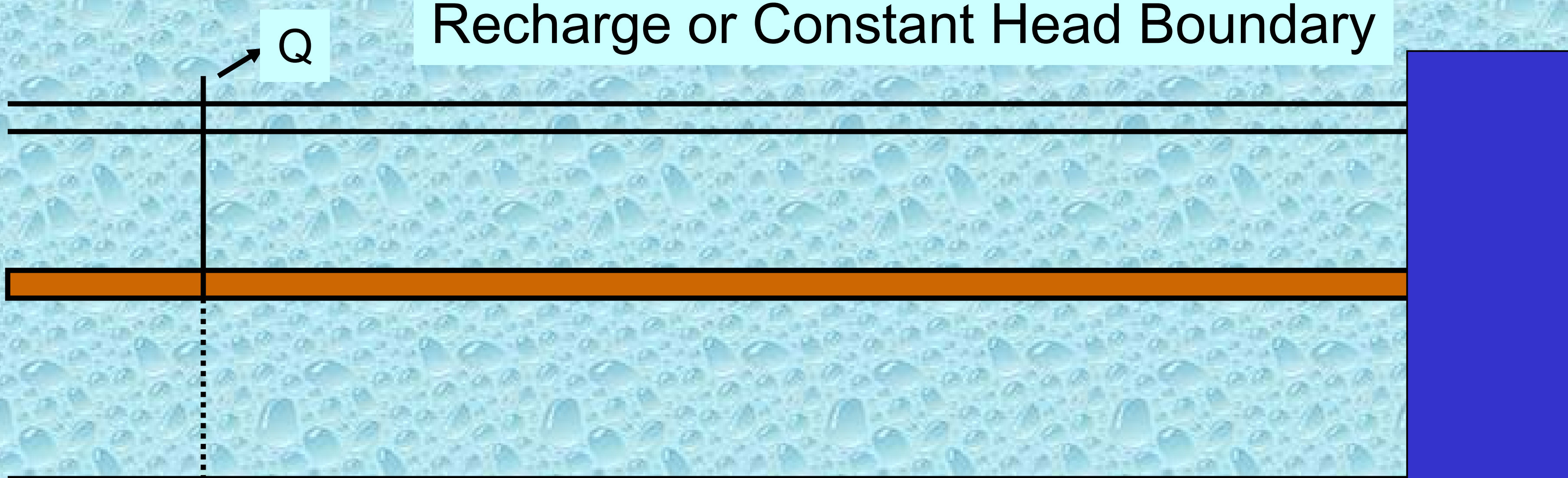
Impact of Boundaries on drawdown as a function of time

No aquifer is infinite. How will boundaries affect response?

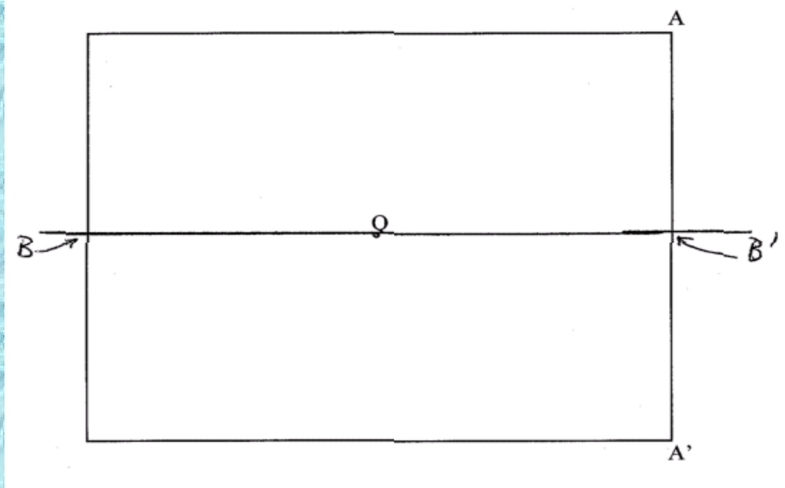
Impermeable or No-flow Boundary



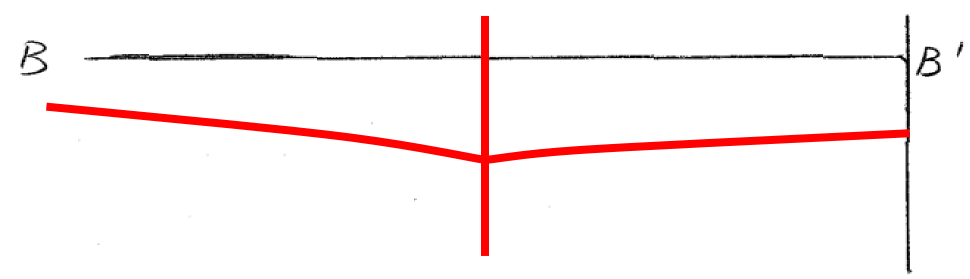
Recharge or Constant Head Boundary



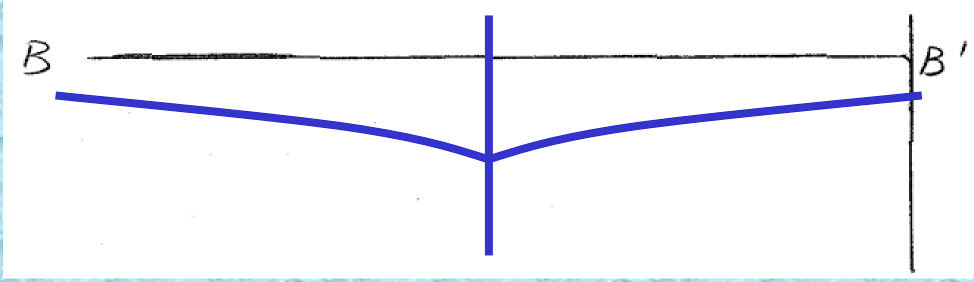
For the following situation with pumping well Q make your qualitative estimates of the relative drawdown.



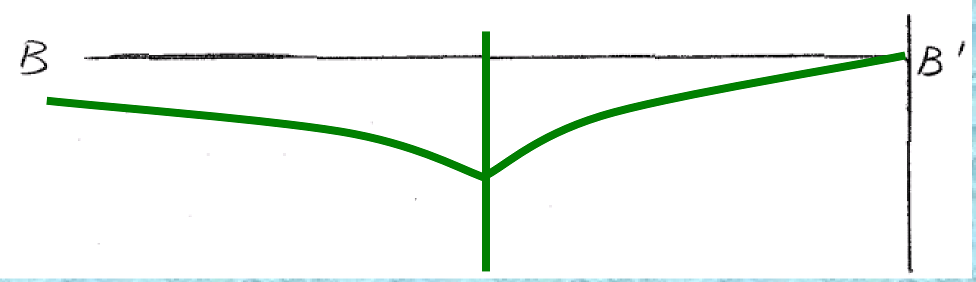
Sketch the cone of depression due to pumping of well Q assuming A-A' is a no flow boundary

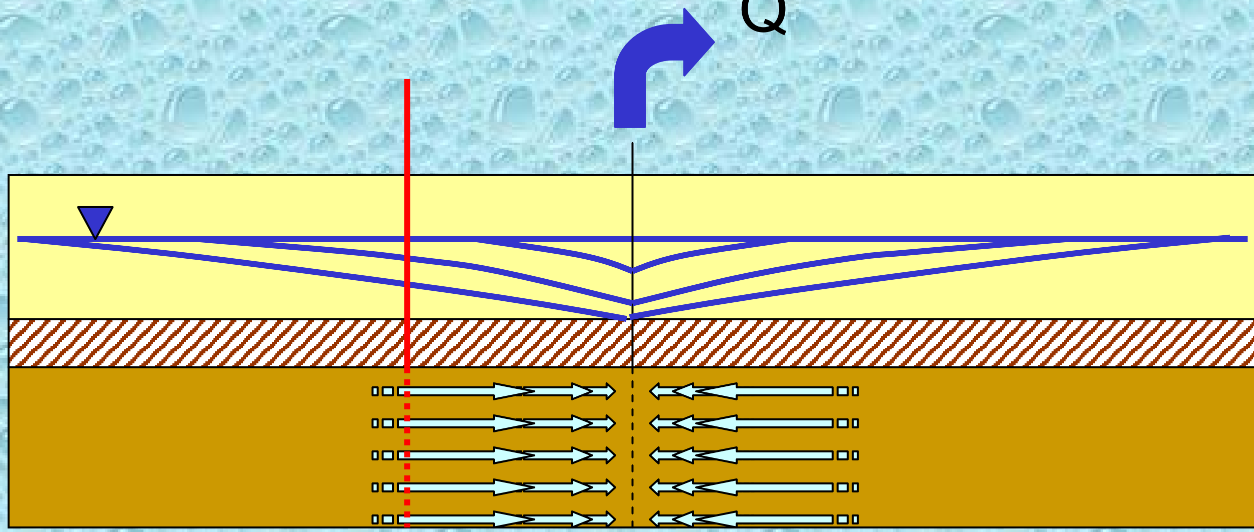


Sketch the cone of depression due to pumping of well Q assuming the aquifer is infinite

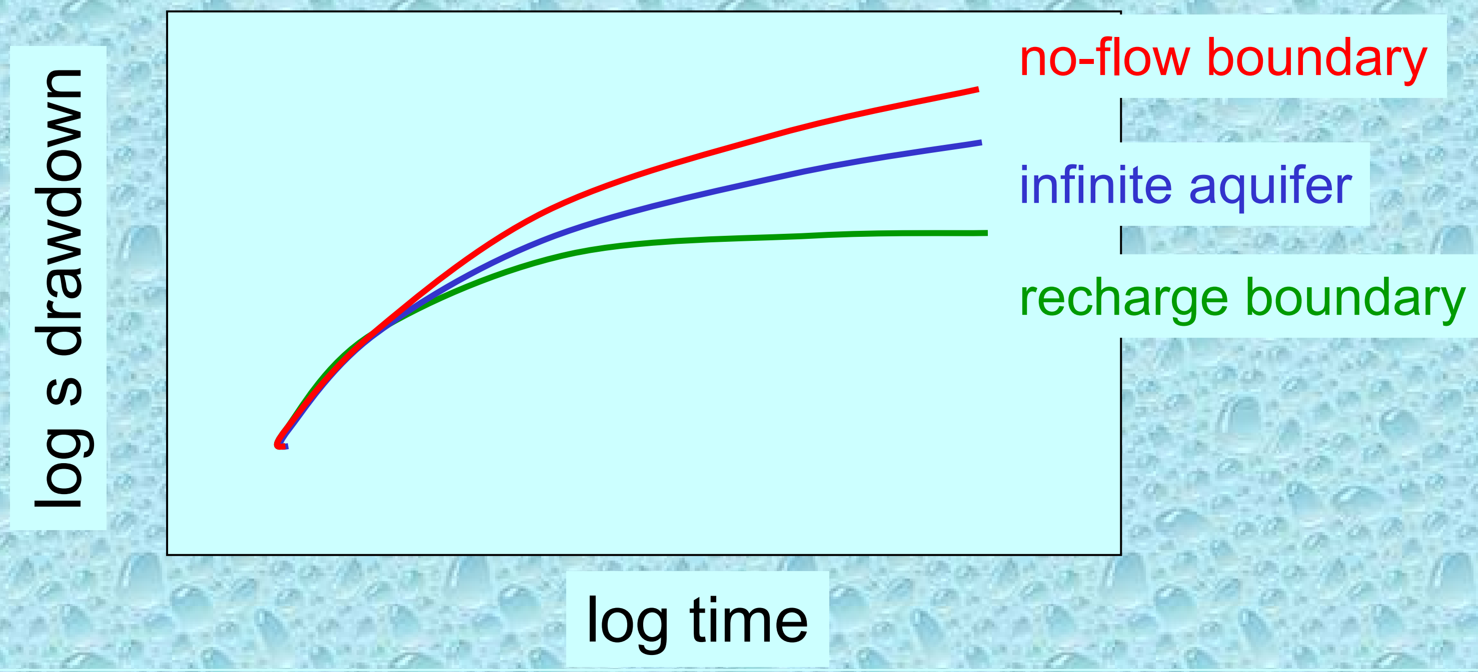


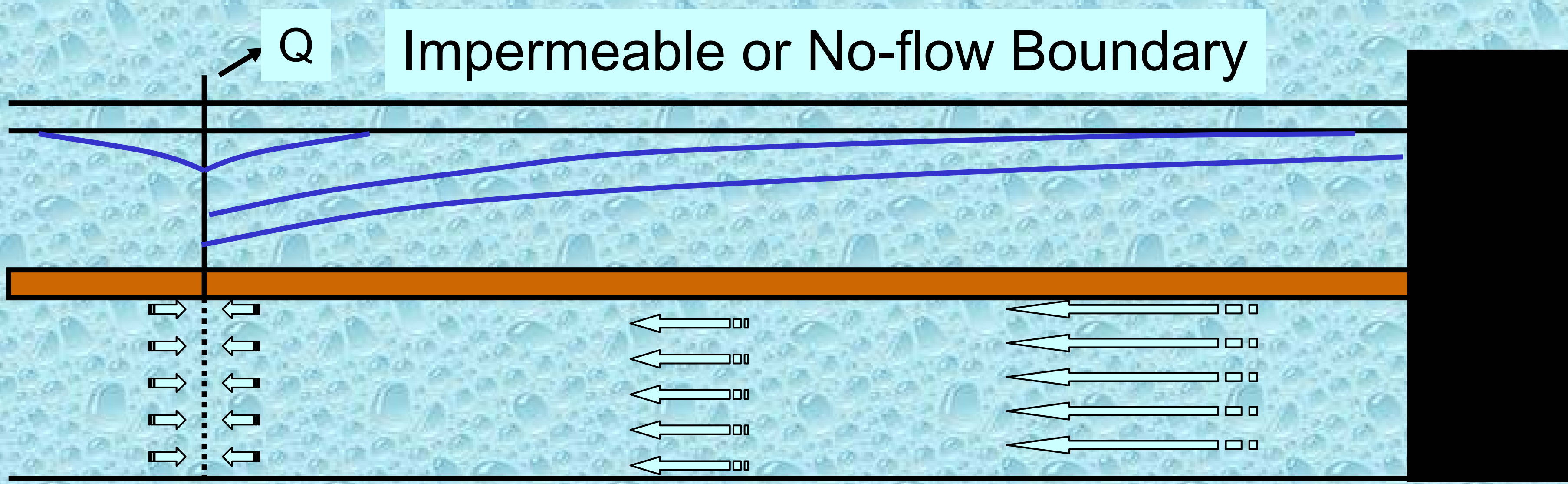
Sketch the cone of depression due to pumping of well Q assuming A-A' is a constant head boundary





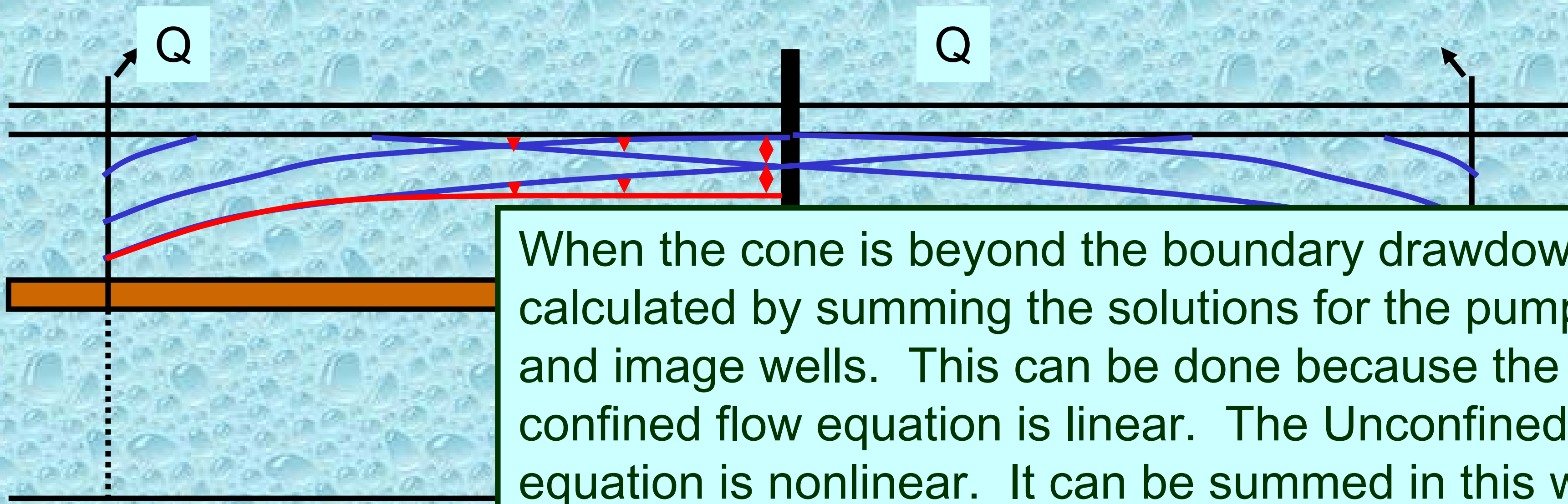
at the red observation well …..





When the drawdown cone reaches the boundary water cannot be drawn from storage in the infinite aquifer, so drawdown occurs more rapidly within the finite aquifer

Impermeable or No-flow Boundary



When the cone is beyond the boundary drawdown is calculated by summing the solutions for the pumping and image wells. This can be done because the confined flow equation is linear. The Unconfined flow equation is nonlinear. It can be summed in this way provided drawdown is relatively small.

Method of Images - can be used to predict drawdown by creating a mathematical no-flow boundary

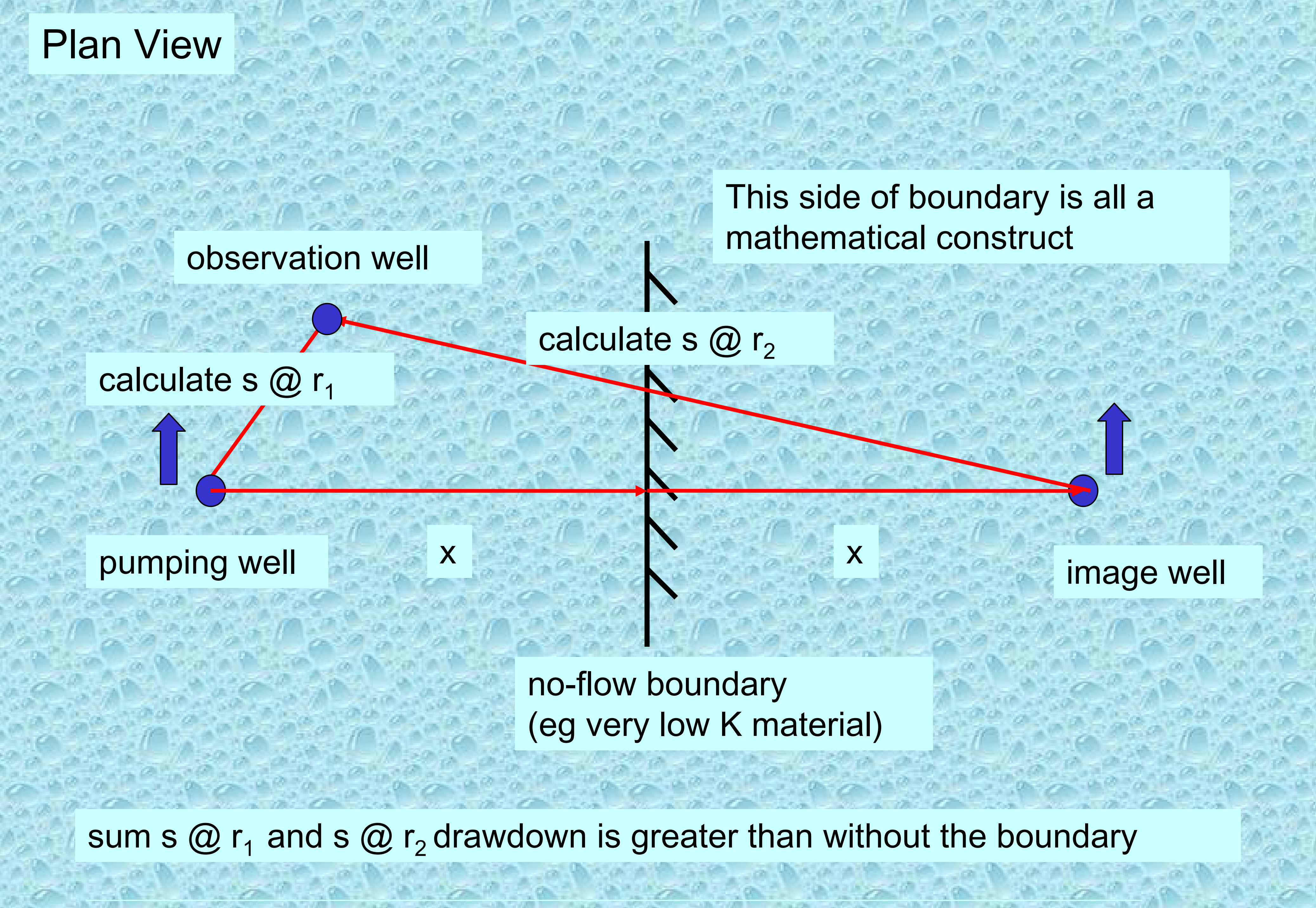
NO-FLOW = NO GRADIENT

So if we place an imaginary well

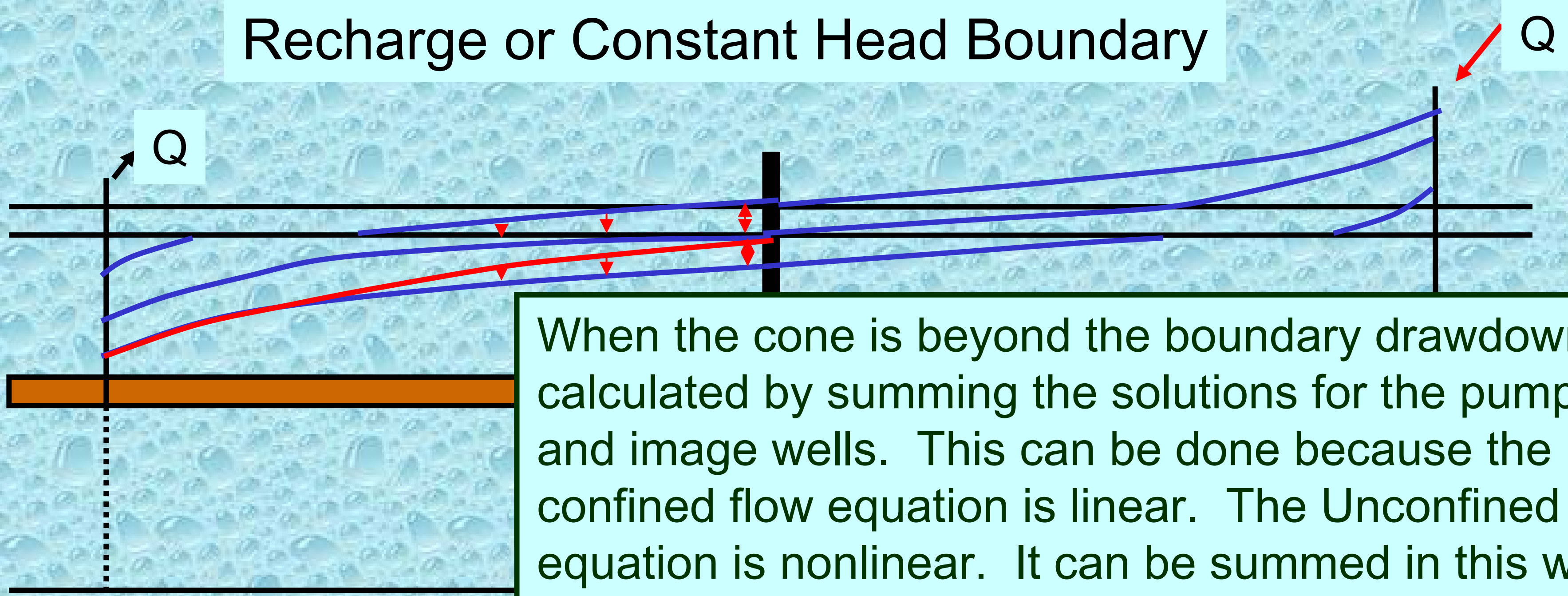
of equal strength

at equal distance across the boundary

and superpose the solutions, we will have equal drawdown, therefore equal head at the boundary, hence NO GRADIENT


Plan View
This side of boundary is all a mathematical construct
observation well
calculate s @ r_2
calculate s @ r_1
pumping well
x
x
image well
no-flow boundary
(eg very low K material)
sum s @ r_1 and s @ r_2 drawdown is greater than without the boundary

Recharge or Constant Head Boundary



When the cone is beyond the boundary drawdown is calculated by summing the solutions for the pumping and image wells. This can be done because the confined flow equation is linear. The Unconfined flow equation is nonlinear. It can be summed in this way provided drawdown is relatively small.

Method of Images - can be used to predict drawdown by creating a mathematical constant head boundary

CONSTANT HEAD = NO CHANGE IN HEAD

So if we place an imaginary well

of equal strength but opposite sign

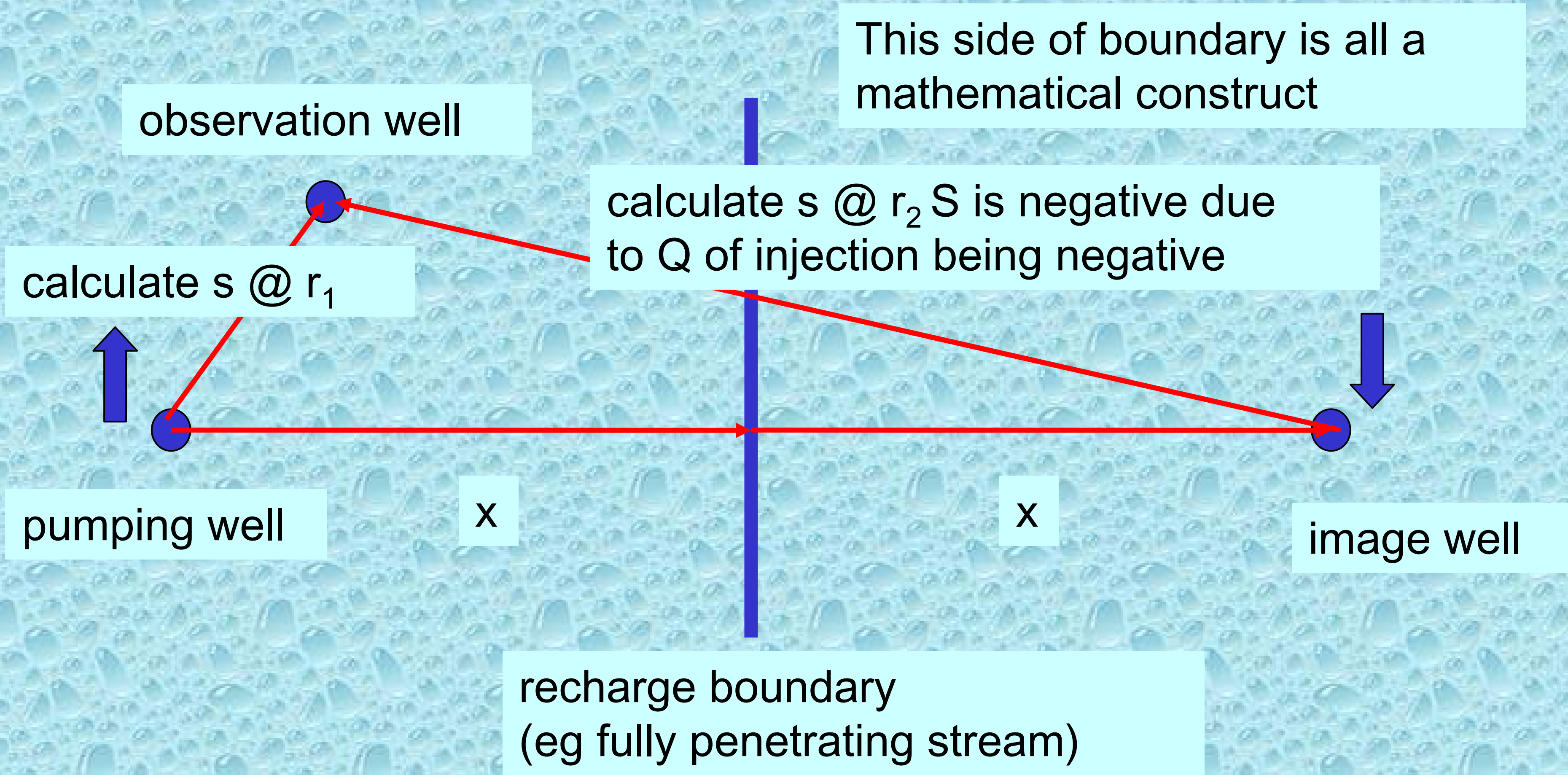
at equal distance across the boundary

And superpose the solutions

We will have

equal but opposite drawdown, therefore NO HEAD CHANGE

Plan View



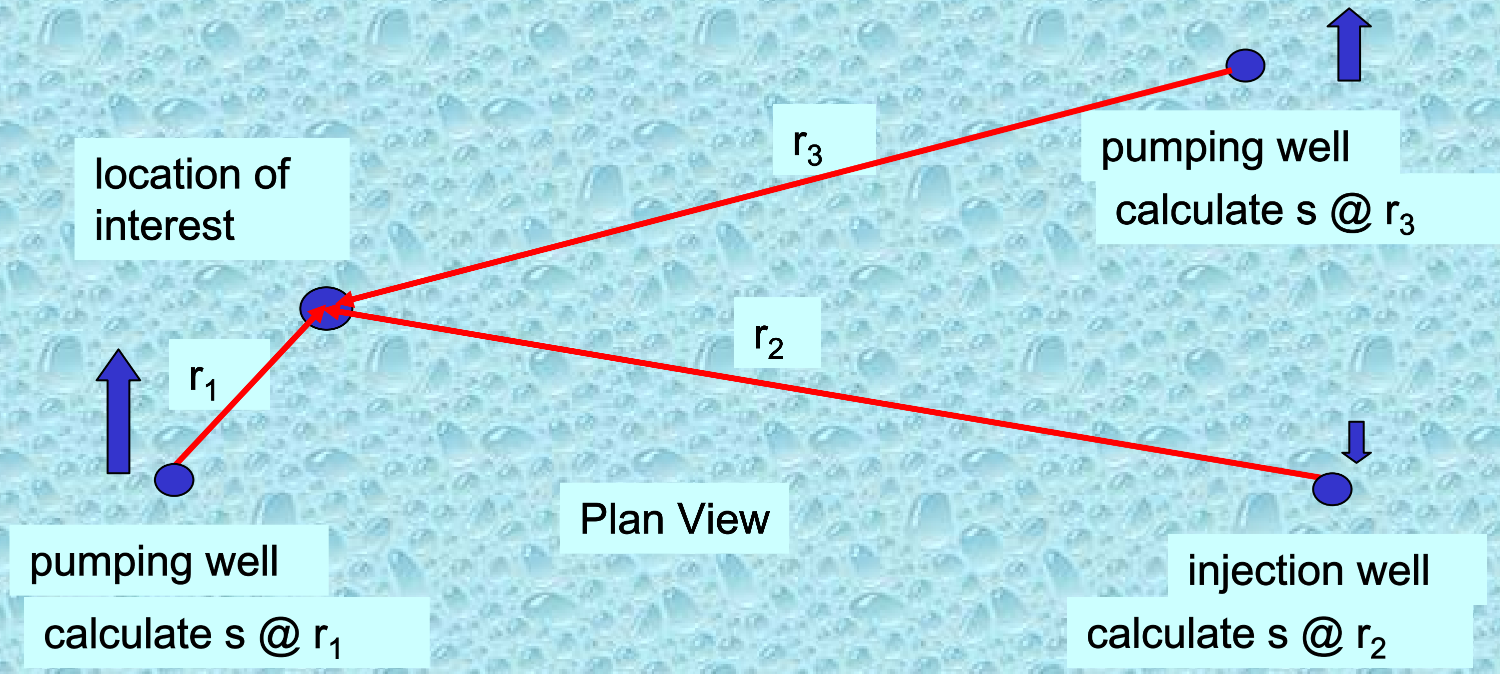
sum s @ r_1 and s @ r_2 drawdown is less than without the boundary

PUMPING FROM A NUMBER OF WELLS

$$s = \frac{Q_1}{4\pi T} W(u_1) + \frac{Q_2}{4\pi T} W(u_2) + \ldots\ldots$$

where:

$$u_i = \frac{r_i^{\,2} S}{4 T t_i} \qquad i = 1, 2, \ldots.$$



sum s_1 from Q_1@ r_1 s_2 from Q_2@ r_2 (note Q_2 is negative) s_3 from Q_3@ r_3 ….. etc etc ….. yields total s at observation well

INCREMENTAL PUMPING

$$s = \frac{Q_1}{4\pi T} W(u_1) + \frac{\Delta Q_2}{4\pi T} W(u_2) + \frac{\Delta Q_3}{4\pi T} W(u_3) \;\ldots\ldots\ldots$$

Q_1 = initial rate — u_1 for t since pumping started, t_1

$\Delta Q_2 = Q_2 - Q_1$ — u_2 for t since incremented rate, t_2

$\Delta Q_3 = Q_3 - Q_2$ — u_3 for t since second increment, t_3

pumping starts at Q_1

start

t=0

t_1

pumping changes to Q_2,calculate s for $Q_2 - Q_1$

t_2

pumping changes to Q_3 ,calculate s for $Q_3 – Q_2$

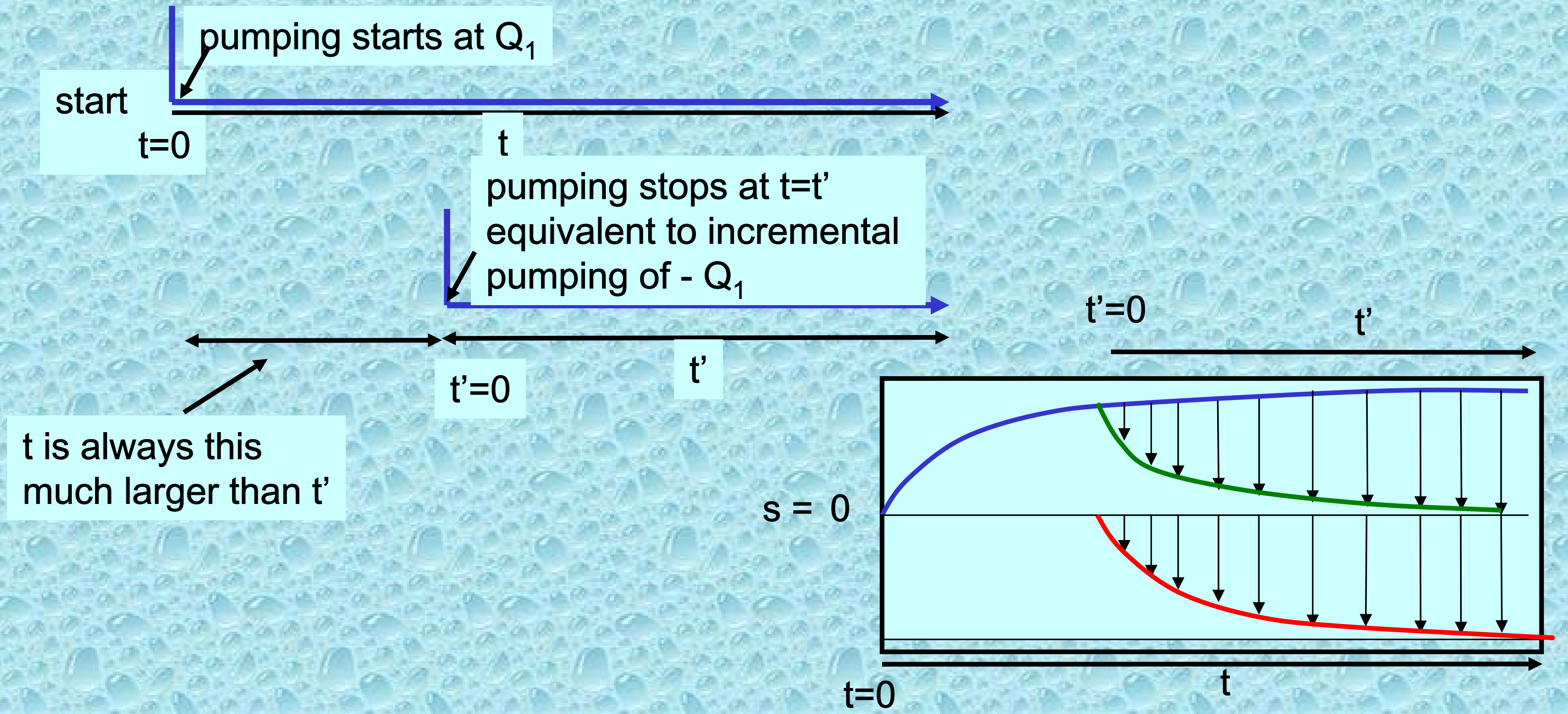
t_3

The sum of all three calculations yields s at t_1 for the varying flow rate

AQUIFER TEST RECOVERY DATA

adding drawdown from injection of –Q at the time when the pump is shut off

$$s' = \frac{Q}{4\pi T}W(u) - \frac{Q}{4\pi T}W(u') \quad \textbf{for} \quad u = \frac{r^2 S}{4Tt} \qquad u' = \frac{r^2 S}{4Tt'}$$



For small u (small r, long t) the Cooper-Jacob relation can be used:

$$s' = \frac{2.3Q}{4\pi T}[\log\frac{2.25Tt}{r^2S} - \log\frac{2.25Tt'}{r^2S}] = \frac{2.3Q}{4\pi T}\log\frac{t}{t'}$$

Plot of s’ vs log(t/t’) is a straight line

$$T = \frac{2.3Q}{4\pi\Delta s}$$

Δs over one log cycle t/t'